Cox-Ingersoll-Ross Interest Rate Calibration and the Pricing of Residential Mortgages

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Abstract

Pricing a mortgage involves discounting a series of cash flows over time by the prevailing interest rates. Since interest rates fluctuate over time, the best way to accomplish this is to choose an appropriate interest rate model and calibrate it to market data. The estimated parameters can then be applied to an amortization/prepayment model which can then be used to calculate the price and/or yield of an individual mortgage or mortgage pool. Calibration is often mentioned in the literature, and while many authors will show its results, the procedure used to estimate these parameters from market data is very seldom laid out in sufficient detail to be of use to a practitioner. In this paper we will calibrate the Cox-Ingersoll-Ross interest rate model to both Corporate and Treasury data between 2005 and 2008. We will map out the procedure that we used and illustrate why we chose a particular model or data set. These decisions include the choice of interest rate model, whether to use short rates and/or forward rates, and whether to use Treasury or Corporate data.

Pricing a Mortgage – Three Sources of Risk

Pricing a mortgage involves dealing with three independent sources of uncertainty or risk--the interest rate, the exogenous prepayment rate, and the default rate. Endogenous prepayments (prepayments result from refinancing, often due to better interest rates) are negatively correlated with interest rates, but exogenous prepayments (due to sale of the house which may occur because of a job change, divorce, etc.) can safely be assumed to be independent of interest rates. We also assume for
simplification purposes that defaults are independent of interest rates. Default rates are related to house prices which we assume to be independent of interest rates (although this is not entirely true as low interest rates typically stimulate demand for real estate). The price of Treasuries involves only interest rate risk, since they are non-callable and backed by the full faith of the U.S. government.

Risks in models for agency mortgages (those issued through Fannie Mae and Freddie Mac) historically omitted defaults because of presumed government protection for investors. Mortgage models that do not involve defaults must be compared a benchmark of comparable credit risk, typically single A corporate bond yields. Including defaults in the model allows us to use risk-free Treasury rates as the benchmark because the losses are built into the cash flows.

**Choosing a Benchmark Interest Rate**

In the prepayment only model, the prepayment option is added to the discount rate which is comparable to the Corporate “A” benchmark with the same duration according to Kalotay [6]:

\[
\text{Mortgage Rate} = \text{Corporate “A” Rate} + \text{Prepayment Option}
\]

By including defaults, in addition to the prepayment option, we add a loss premium which can be calculated from the reduced cash flows. Thus we can use the risk-free rate with the same duration (Treasury rate) as a base; the prepayment option and the loss premium can be added to obtain the mortgage rate:

\[
\text{Mortgage Rate} = \text{Risk-Free Rate} + \text{Prepayment Option} + \text{Loss Premium}
\]

Since we use the short rate in our model, instead of the calibrating the interest rate model to the Corporate A data, we can calibrate it directly to U.S. treasury rates. This is a distinct advantage
since treasury issues are seen as a purer benchmark than corporate bonds and they are also more extensively archived.

**The interest rate model**

The simplest short-rate model that can be calibrated to market data is Ho-Lee which consists of a deterministic, time-dependent part and a stochastic part. It is represented by the following differential equation:

\[ dr_t = \Theta dt + \sigma dW_t \]  

(1.1)

where \( \Theta \) and \( \sigma \) are parameters, and \( W_t \) is a Brownian motion process.

The first mathematical model to describe the random fluctuations of interest rates using mean-reversion was the one introduced in 1977 by Oldrich Vasicek [8]. It is a one-factor, short-rate model as it describes interest rate movements as driven by only one source of market risk. The model can be used in the valuation of interest-rate derivatives. The main shortcoming of this model is that it implies the existence of negative interest rates and is therefore not useful in the valuation of credit instruments.

One can write a stochastic differential equation that describes the model as:

\[ dr_t = \kappa (\theta - r_t) dt + \sigma dW_t \]  

(1.2)

Under the risk-neutral measure, the parameter \( \theta \) is the long-term mean interest rate, \( \sigma \) is the volatility term, and \( \kappa \) represents the rate of adjustment of the mean-reversion.

Introduced in 1985, the Cox-Ingersoll-Ross model [2] also describes the evolution of interest rates. The stochastic differential equation that captures the essence of this model is

\[ dr_t = \kappa (\theta - r_t) dt + \sigma \sqrt{r_t} dW_t \]  

(1.3)

The factor \( \sqrt{r_t} \), introduced in the stochastic term, makes the model more complicated due to its non-linearity but also more realistic. If \( 2\kappa \sigma \geq \sigma^2 \), the interest rate cannot become negative.
In order to find $\theta$, $\kappa$ and $\sigma$ for the Cox-Ingersoll-Ross (CIR) model, we can obtain historical short rates or express the theoretical forward rates via the affine model for bond pricing. We can approximate short rates by looking at historical one-month treasuries for example. The problem with short rates is that we can only obtain them from the past. Forward rates, on the other hand, project future interest rates. Thus if we wish to predict the direction of interest rates it makes sense to examine forward rates. Empirical forward rates must reflect the credit risk of residential mortgages; these can be obtained from using as a benchmark a single A corporate bond. (See Kalotay[6], and Gorovoy and Linetsky[4]). Yields of such bonds are provided by Bloomberg. By transforming yields into forward rates and then applying the least squares method, we can fit the theoretical forward rate to that empirical forward rate data. The procedure boils down to a non-linear, non-convex minimization problem which can be addressed by an appropriate solver.

**Estimating Parameters From Short Rates**

Although we cannot obtain future short rates, we can look at historical short rates because they are more intuitive than forward rates. We can estimate the long-term CIR rate by simply taking the average of the historical short rates:

$$
\theta = E[r_t] = \frac{1}{n} \sum_{i=1}^{n} r_i
$$

From this we can estimate $\sigma$ using the following straightforward estimation method:

$$
dr_t = \kappa (\theta - r_t) dt + \sigma \sqrt{r_t} dW_t,
$$

$$
E[dr_t] = \kappa (\theta - r_t) dt
$$

$$
dr_t - E[dr_t] = \sigma \sqrt{r_t} dW_t
$$

$$
(dr_t - E[dr_t])^2 = \sigma^2 r_t dt
$$

$$
\sqrt{\text{var}[dr_t]} = \sigma
$$

Thus our estimate for the volatility is:
\[ \hat{\sigma} = \frac{\text{var}[\Delta r]}{\sqrt{E[\Delta r] \Delta t}} \]  

(1.5)

Using 1-month treasury rates collected monthly for the five-year period from 11/1/2000 until 10/1/2005, we obtain the following estimates: \( \theta = 0.02163 \) and \( \sigma = 0.05547 \). Solving for \( \kappa \) using short rates remains a subject for future research. Remembering that these parameters are based on historical data, we still find this to be a useful method to choose a suitable starting point for the optimizer. But we can look into the future by examining forward rates based on the term structure of interest rates.

**Optimization and Calibration Using Forward Rates**

Calibration involves selecting parameters which best fit the actual data. This can be defined as a non-linear, least-squares optimization problem. We define the objective function as:

\[
\min \sum_{i=1}^{n} \left[ f(t_i, T_i) - \hat{f}(t_i, T_i; \theta, \kappa, \sigma) \right]^2 \quad \text{subject to} \quad \theta, \kappa, \sigma > 0
\]

(1.6)

where \( f(t_i, T_i) \) is the empirical forward rate obtained from Bloomberg at time \( t \), and \( \hat{f}(t_i, T_i; \theta, \kappa, \sigma) \) is the theoretical forward rate expressed in terms of the model parameters \( \theta, \kappa \) and \( \sigma \) that have to be estimated. To express the theoretical forward rate, recall that in the affine model, the bond price

\[
B(t, T, r_t) = e^{-\beta(t-T) - C(t, t)\rho_t}
\]

(1.7)

The functions \( A(t, T) \) and \( C(t, t) \) can be found in [7] and they are:

\[
A(t, T) = -\frac{2k\theta}{\sigma^2} \ln \left( \frac{\rho e^{\Delta s(T-t)}}{\rho \cosh \left[ \frac{1}{2} \rho (T-t) \right] + \kappa \sinh \left[ \frac{1}{2} \rho (T-t) \right]} \right)
\]

(1.8)

and

\[
C(t, T) = \frac{2 \sinh \left[ \frac{1}{2} \rho (T-t) \right]}{\rho \cosh \left[ \frac{1}{2} \rho (T-t) \right] + \kappa \sinh \left[ \frac{1}{2} \rho (T-t) \right]}
\]

(1.9)
where \( \rho = \sqrt{\kappa^2 + 2\sigma^2} \). Since the forward rate is:

\[
    f(t, T) = -\frac{\partial \ln B(t, T)}{\partial T} = \frac{\partial A}{\partial T} + \frac{\partial C}{\partial T} r_t
\]

(1.10)

and we have

\[
    \frac{\partial A}{\partial T} = \frac{2\kappa\theta \sinh \left[ \frac{1}{2} \rho (T-t) \right]}{\rho \cosh \left[ \frac{1}{2} \rho (T-t) \right] + \kappa \sinh \left[ \frac{1}{2} \rho (T-t) \right]}
\]

(1.11)

\[
    \frac{\partial C}{\partial T} = \frac{\rho^2}{\left( \rho \cosh \left[ \frac{1}{2} \rho (T-t) \right] + \kappa \sinh \left[ \frac{1}{2} \rho (T-t) \right] \right)^2}
\]

(1.12)

it follows that we can rewrite (1.10) as

\[
    f(0, T) = \frac{2\kappa\theta \sinh \left( \frac{1}{2} \rho T \right)}{G(T)} + \left[ \frac{\rho}{G(T)} \right]^2 r_0
\]

where \( G(T) = \rho \cosh \left( \frac{1}{2} \rho T \right) + \kappa \sinh \left( \frac{1}{2} \rho T \right) \)

(1.13)

**A Solver for Non-Linear Optimization**

Gorovoy and Linetsky [4] suggest using Benchmark Corporate A-rated bonds to match the credit risk of residential mortgages. From Bloomberg we can obtain the Implied Forwards Curve from yields of Composite Corporate A Benchmark as well as for treasury bonds depending upon whether we choose the prepayment only model or the prepayment model with defaults. Once we have obtained the forward rates, we can use non-linear optimization to obtain a solution to (1.6). We use unconstrained minimization except for non-negativity constraints applied to the parameters \( \theta, \kappa \) and \( \sigma \). We were able to obtain identical solutions using two different solvers: KNITRO and the NMinimize function in Mathematica.
KNITRO

These empirical rates are assigned to the parameter “fwd” listed in the AMPL source code below which can be used to find a local minimum. The source code can be run under the NEOS Solver for optimization and can be run from the web site: http://www-neos.mcs.anl.gov/

```
param N := 51;
param fwd{1..N};
param T{i in 1..N} := i/4;
param r0 := 0.0395;
var theta >= 1e-6;
var kappa >= 1e-6;
var sigma >= 1e-6 default 0.038988537;
var rho = sqrt(kappa^2+2*sigma^2);
var G{i in 1..N} = rho*cosh(0.5*rho*T[i])+kappa*sinh(0.5*rho*T[i]);
var A{i in 1..N} = 2*kappa*theta*sinh(0.5*rho*T[i])/G[i];
var C{i in 1..N} = rho/G[i];
var F{i in 1..N} = 100*(A[i]+r0*C[i]^2);
minimize lsq: sum {i in 1..N}(fwd[i]-F[i])^2;
```

The solvers Couenne [1] and/or KNITRO can be used to perform nonlinear optimization. Both KNITRO and Couenne takes input the AMPL source listed above [3] along with the forward rates obtained from Bloomberg. Couenne can solve non-convex problems using a branch-and-bound approach; KNITRO is much quicker but only finds a local minimum. If we choose a suitable starting point, KNITRO can find the optimal value.

MATHEMATICA

The Mathematica Source used to estimate CIR parameters is listed below. The function NMinimize always attempts to find a global minimum numerically. The forward rates are stored in the variables starting with FC05M1, the naming conventions of which are listed below:

```
(* Naming Convention for forward rates:       FBYYMI
 F = Forward Rate
 B = Bond Type:  C = Corporate, T = Treasury
 YY = Last two digits of year (Starting in October)
 M = Interval Units (Months)
 I = Interval 1 or 3 months
 e.g. FC05M1 contains the 1-month Corporate Forward Rates for Oct 2005 *)
```
\[ \begin{align*}
F_1 &= \{FC05M1, FT05M1, FC06M1, FT06M1, FC07M1, FT07M1, FC08M1, FT08M1\} \\
F_3 &= \{FC05M3, FT05M3, FC06M3, FT06M3, FC07M3, FT07M3, FC08M3, FT08M3\} \\
R_0 &= \{3.95, 3.485, 6.775, 4.9222, 4.036, 6.1337, 4.036, 3.6779, 0.7042\} \\
J &= B + 2(Y - 5) \\
r_0 &= R_0[J]/100 \\
FWD &= \{F_1, F_1, F_3\}[\text{INT}][J] \\
TIME &= \text{Range}[\text{Length}[\text{FWD}]]/(12/\text{INT}) \\
\rho &= \sqrt{\kappa^2 + 2\sigma^2} \\
A &= 2\kappa\theta\text{Sinh}[0.5\rho\text{TIME}]/G \\
G &= \rho\text{Cosh}[0.5\rho\text{TIME}] + \kappa\text{Sinh}[0.5\rho\text{TIME}] \\
\CC &= \rho/G \\
F &= 100(A + r_0\CC^2) \\
dt &= .000001 \\
\text{INT} &= 1 \\
Y &= 5 \\
B &= 1 \\
\{Y+2000, \text{INT}, B\} \\
\text{NMinimize}\{\text{Total}[(\text{FWD} - F)^2], \kappa \geq dt, \theta \geq dt, \sigma \geq dt\}, \{\kappa, \sigma, \theta\}\}
\end{align*} \]

Calibration to Corporate and Treasury Benchmarks

From both the KNITRO solver from NEOS and the NMinimize solver from Mathematica, we obtain the following parameters; using the initial short rate \( r_0 = f(0,0) \).

<table>
<thead>
<tr>
<th>DATE</th>
<th>TYPE</th>
<th>INTERVAL</th>
<th>Kappa</th>
<th>Sigma</th>
<th>Theta</th>
<th>Ro</th>
<th>Obj</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/03/2005</td>
<td>CORP</td>
<td>1 month</td>
<td>0.19733</td>
<td>0.07571</td>
<td>0.06126</td>
<td>4.0923</td>
<td>1.31937</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 month</td>
<td>0.19368</td>
<td>0.07372</td>
<td>0.06100</td>
<td>4.14002</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TSY</td>
<td>1 month</td>
<td>0.22757</td>
<td>0.10120</td>
<td>0.05172</td>
<td>3.485</td>
<td>7.58890</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 month</td>
<td>0.21755</td>
<td>0.09683</td>
<td>0.05224</td>
<td>2.44065</td>
<td></td>
</tr>
<tr>
<td>10/03/2006</td>
<td>CORP</td>
<td>1 month</td>
<td>0.34544</td>
<td>0.32488</td>
<td>0.06489</td>
<td>6.775</td>
<td>1.19572</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 month</td>
<td>0.36308</td>
<td>0.39448</td>
<td>0.07121</td>
<td>2.44065</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TSY</td>
<td>1 month</td>
<td>2.03235</td>
<td>1.75419</td>
<td>0.05768</td>
<td>4.9222</td>
<td>3.10067</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 month</td>
<td>1.81568</td>
<td>1.69914</td>
<td>0.05975</td>
<td>1.07442</td>
<td></td>
</tr>
<tr>
<td>10/03/2007</td>
<td>CORP</td>
<td>1 month</td>
<td>0.73564</td>
<td>0.67264</td>
<td>0.06721</td>
<td>6.1337</td>
<td>5.17664</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 month</td>
<td>0.87592</td>
<td>0.81716</td>
<td>0.07025</td>
<td>3.39505</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TSY</td>
<td>1 month</td>
<td>1E-6</td>
<td>1E-6</td>
<td>1202.00000</td>
<td>4.0360</td>
<td>5.18068</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 month</td>
<td>0.00266</td>
<td>0.00026</td>
<td>0.49946</td>
<td>1.73456</td>
<td></td>
</tr>
<tr>
<td>10/03/2008</td>
<td>CORP</td>
<td>1 month</td>
<td>1E-6</td>
<td>1E-6</td>
<td>7307.46</td>
<td>3.6779</td>
<td>19.8954</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 month</td>
<td>1E-6</td>
<td>1E-6</td>
<td>7293.69</td>
<td>6.7927</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TSY</td>
<td>1 month</td>
<td>0.14551</td>
<td>1E-6</td>
<td>0.54152</td>
<td>0.7042</td>
<td>15.0645</td>
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<tr>
<td></td>
<td></td>
<td>3 month</td>
<td>0.00291574</td>
<td>1E-6</td>
<td>2.55966</td>
<td>4.95138</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Estimated CIR Parameters using Calibration to forward rates using \( r_0 = f(0,0) \)

We looked at data from each of the years 2005, 2006, 2007 and 2008 for both Corporate and Treasury data. We decided to look at one-month forward rates in one-month intervals as well as three-month forward rates in three-month intervals—both over the exact same time span. We took 100 data
points from the one-month data and 33 data points from the three-month data so that the parameters were comparable. Notice that the estimates for $\kappa$, $\sigma$ and $\theta$ are fairly close for the one- and three-month forward rates. The values of $\theta$ differ by 94-95 basis points between corporate and treasury bonds for 2005. It is reasonable that the long-term mean of Corporate A bonds should be higher than that of the Treasuries. Since these parameters are estimated using the risk-neutral measure, we cannot read any special meaning into specific values except perhaps the volatility $\sigma$ which is not sensitive to the change of measure.

In 2005 and 2006, the parameters are relatively stable, but in 2007 and 2008, the optimal values for the parameters appear to be far outside the normal range; however, the theoretical curves calculated from these parameters appear to fit the empirical data. The charts below and on the following page show the theoretical and actual forward rate curves for both Treasuries and Corporate A benchmarks for Oct 3 in each of the years 2005, 2006, 2007 and 2008. The theoretical curves use the estimated three-month parameters in Table 1. Note that 2005 is well behaved, but in 2006, the yield curve is inverted, and in 2007 the Corporate curve decreases while the treasury curve increases. In 2008, the forward curve appears to be both linear and increasing, although the lower bound constraint for $\sigma$ appears to be close to zero and binding. Until economic conditions stabilize, calibration of interest rates may prove to be difficult if not impossible.
The Initial Instantaneous Short Rate

Forward rates are measurable from a particular point in time; however, the instantaneous short rate is not directly measurable. Gorovoy [4] most likely took the shortest possible rate (monthly or weekly), or even did some interpolation to daily. For the October 2005 forward rates he uses the value of 3.95% for the short rate instead of the one-month rate of 4.0923 reported by Bloomberg. Since the initial short rate must be estimated, it makes sense to analyze the sensitivity of the parameters to changes in the initial short rate. The parameters for the Corporate A Benchmark when the initial short rate is set to 3.95% are as follows:

<table>
<thead>
<tr>
<th>DATE</th>
<th>TYPE</th>
<th>INTERVAL</th>
<th>Kappa</th>
<th>Sigma</th>
<th>Theta</th>
<th>Ro</th>
<th>Obj</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/3/2005</td>
<td>CORP A</td>
<td>1 Month</td>
<td>0.247863</td>
<td>0.108839</td>
<td>0.0609254</td>
<td>3.95</td>
<td>1.84746</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 Month</td>
<td>0.240924</td>
<td>0.105575</td>
<td>0.0613137</td>
<td></td>
<td>0.565517</td>
</tr>
</tbody>
</table>

For the 2005 data, observe that if the short rate for the Corporate A benchmark is between 3.5% and 4.16%, the parameters fall into a fairly narrow range, exhibiting linear behavior. (See Figure 3). For the treasury data in Figure 4, we observe similar behavior between the short rate of 3.4% and 3.7%. For the 2006 corporate A benchmark, the parameters are not nearly as robust as seen in Figure 5. Observe that at 6.75% which is near the initial short rate, $\kappa$ and $\sigma$ are

![3-Month Forward Rates 10/03/2008](chart.png)
almost equal to each other, but at 5.45%, the values are very far apart. Notice also that \( \theta \) gets very large as the initial short rate moves below 5.5%. It appears that if we are dealing with well-behaved data, our model works well as long as the initial short rate is within a reasonable range, but if the parameter estimates are out of the normal range, they will also be extremely sensitive to the initial short rate.

Figure 3

Sensitivity of Parameters to Current Short Rate
(1-mo CORP 10/3/05)

![Graph showing sensitivity of parameters to current short rate for 1-mo CORP 10/3/05. The graph displays the relationship between short rate and parameters Kappa, Sigma, and Theta.]  

Figure 4

Sensitivity of parameters to Current Short Rate
(1-mo TSY 10/3/05)

![Graph showing sensitivity of parameters to current short rate for 1-mo TSY 10/3/05. The graph displays the relationship between short rate and parameters Kappa, Sigma, and Theta.]
Summary

Since one of the main goals of this paper is to define a procedure for the calibration of CIR parameters from empirical data, we present it here in summarized form:

1. Obtain 3-month Benchmark Corporate A Rates and the corresponding forward rates from Bloomberg.
2. Estimate long-term average interest rate
   \[ E[r] = \frac{\sum_{i=0}^{n} r_i}{n} \]
3. Calculate
   \[ \text{var}[\Delta r] = \frac{\sum_{i=1}^{n} (\Delta r_i - \bar{\Delta} r)^2}{(n-1)}, \Delta r_i = r_i - r_{i-1} \]
4. Estimate
   \[ \hat{\sigma} = \sqrt{\frac{\text{var}[\Delta r]}{E[r] \Delta t}} \] (Note \( \Delta t = 1/12 \) for monthly rates or \( 1/252 \) for daily rates).
5. Set \( \rho = \sqrt{\kappa^2 + 2\sigma^2} \) and \( G(T) = \rho \cosh \left( \frac{1}{2} \rho T \right) + \kappa \sinh \left( \frac{1}{2} \rho T \right) \)
6. Now given forward rates and the current short rate find \( \theta, \kappa, \) and \( \sigma \), using the previous results as a starting point:
\[
\min \sum_{i=1}^{2} \left[ f(0, T_i) - \frac{2 \kappa \theta \sinh \left( \frac{1}{2} \rho T_i \right)}{G(T_i)} + \left( \frac{\rho}{G(T_i)} \right)^2 r_0 \right]^2
\]

**Conclusion**

Using Treasury forward rates instead of those calculated from the Benchmark Corporate A Bonds has potential for use in mortgage models which takes defaults into account. This allows one to model the mortgage rate as a sum of the Treasury Rate plus the Option Adjusted Spread for prepayments plus the loss premium for defaults. This loss premium is similar to the risk premium in the Capital Asset Pricing Model (CAPM).

The simple approach of calculating the value of \( \theta \) as the average of historical interest rates, and then calculating \( \sigma \) using the straightforward method is useful as a starting point for parameter estimation, but it is limited because it is looking backward rather than forward. Forward rates take advantage of the term structure of interest rates to look into the future.

In stable years like 2005, calibration of parameters using forward rates is fairly consistent, but it becomes more difficult under unusual economic conditions such as the recent economic meltdown. The parameters are also robust to the initial short rate in stable economic times, but become less so in recent years for the same reason. The risk-neutral measure used in the short-rate model prevents us from assigning any meaning to particular parameter estimates except perhaps \( \sigma \) which does not depend upon the change of measure. This makes it difficult to interpret the results of the calibration. Comparing actual to theoretical forward rates graphically is the best way to determine how well the model fits.

In the paper we have put together a procedure to calibrate Cox-Ingersoll-Ross parameters. Once the parameters have been established, we can use them to discount the expected mortgage cash flows to price an individual mortgage or a mortgage pool.
References