3. BASICS OF CAPITAL BUDGETING

Objectives: After reading this chapter, you should be able to
1. Apply the Net Present Value rule to determine acceptable projects for a corporation.
2. Calculate the impact of depreciation and taxes on the NPV of a project.

3.1 Capital Budgeting

One of the most important decisions at a corporation is the investment decision. This involves the evaluation of projects and separation of the profitable projects from the unprofitable ones. A corporation can increase its earnings and its total value by investing in profitable projects. A company has to decide whether to build a new factory, or simply to drop the idea. They have to decide whether to buy a warehouse or to lease it. If they are considering two possible methods of providing air conditioning in a building, they have to decide on one that costs less in the long run. The company wants to evaluate the profitability of an investment in machinery and equipment. The basic question is to find out whether a given project is worthwhile or not.

Capital budgeting deals with the long-term investment decisions of a firm. The profitable projects are accepted and the others rejected. Sometimes more than one project can be profitable and thus it is necessary to rank projects in terms of their desirability so that the company may select the best ones. This is because the capital is difficult to obtain and it is necessary to use it in the most productive way.

There are several methods of ranking profitable projects, but by far the best one is the Net Present Value method. The net present value compares the initial investment in the project with the present value of future cash flows using an appropriate discount rate. We define the net present value, NPV, as follows:

$$\text{NPV} = -I_0 + \sum_{i=1}^{n} \frac{C}{(1 + r)^i}$$

(3.1)

Here, $I_0 =$ the initial investment in the project
$C =$ the cash flow from the project
$r =$ risk-adjusted discount rate
$n =$ life of the project in years

The method essentially does a cost-benefit analysis. The first term on the right side of (3.1) is the cost of the project, and the second term is the present value of the benefits. It is important to know that projects vary greatly in their degree of risk. Therefore, it is necessary to use a risk-adjusted discount rate to evaluate them. The riskier projects will have a higher discount rate.
The net present value is an extremely powerful concept in finance. It is applicable to a variety of problems, including the investment analysis of projects. The acceptability criterion is quite simple: the projects with positive NPV are profitable and the management will accept them. The negative NPV projects are not acceptable because they will reduce the value of the corporation.

An alternative method for evaluating projects is to find their *internal rate of return (IRR)*. The IRR of a project is that particular discount rate, which makes the NPV of a project equal to zero. To find IRR, we set

\[
NPV = 0 = -I_0 + \sum_{i=1}^{n} \frac{C_i}{(1 + r)^i}
\]

and then solve the above equation for \( r \), which is the desired internal rate of return. The actual calculation of IRR is not always easy. Once we find it, we compare it to the risk-adjusted required rate of return for the project. If IRR is greater, the project is acceptable.

It is also possible to rank various projects by a very simple method, the *payback-period* method. The payback period is the time necessary to recover the initial investment from a project. The shorter the payback period, the more desirable is the project. The use of this method, however, can lead to some serious difficulties and its use is highly questionable.

There are three serious flaws with the payback-period method. First, it fails to take into account the entire set of cash flows. It ignores the cash flows that occur after the initial cost is recovered. Some of these cash flows can be quite large. If they are negative, they can reduce the value of the project.

Second, the payback-period method does *not* look at the time value of money. The concept of discounting, so fundamental in finance, is missing in this method. It considers the $100 made in the first year to be equivalent to the $100 made in the second year.

Third, payback-period method ignores the risk of a project. It does not look into the safety of the cash flows. It does consider the probability of unexpected breakdown of equipment, for example. For these reasons, one should not use the payback method.

Equation (3.1) involves five quantities, NPV, \( I_0 \), \( C \), \( n \), and \( r \). If any four of them are given, one can find the fifth quantity.

### 3.2 NPV Problems

The net present value problems fall into the following categories:

1. *Basic problems*: One can solve these elementary problems by the direct use of equation (3.1). Complications arise due to the following factors.

2. *Uneven cash flow problems*: Equation (3.1) is valid only if the cash flows are constant year after year. If the cash flows do not show a regular pattern, then we must discount
them individually, and then add the discounted amounts. This is frequently the case when the returns from a given investment are rather slow in the beginning, then they increase substantially reaching a plateau, and finally drifting downward again.

3. **Uncertain cash flow problems**: If the cash flow is uncertain, it may be possible to improve the result by assigning some subjective probabilities to the different cash flows. By multiplying each probability to the corresponding cash flow and adding the results, we can calculate the expected cash flow. We then use this expected cash flow to complete the calculation. This is the case when the cash flows are dependent on the economic outlook, weather conditions, or traffic patterns.

3. **Uncertain life of project problems**: This type of a problem stems from the uncertainty in the life of a project. For instance, we do not know how long a machine will last or how soon a particular technology will become obsolete. Once again, we assign each (uncertain) life of the project a subjective probability. Then we multiply each probability by the dollar outcome of the respective life. The sum of these then gives the expected NPV of the project.

5. **Problems involving depreciation and taxes**: Most of the corporations pay income taxes. The corporations also take all available tax deductions before they pay the taxes. Generally, all business expenses, salaries to the employees, cost of goods sold, rent, and interest are tax deductible.

Another very important deduction is the **depreciation** of machinery and equipment. The physical deterioration of the assets reduces their productivity, and hence their value. This decrease in value, calculated according to a standard accounting method, is the depreciation. The Internal Revenue Service recognizes it as a legitimate business expense and allows it as a tax deduction. We must understand that depreciation is a non-cash expense.

Let us define the cash flow as the earnings after taxes. We can write \( C \) in (3.1) as follows:

\[
\text{Cash flow} = \text{Earnings before taxes} - \text{taxes} \\
= \text{Earnings before taxes} - (\text{income tax rate}) \times (\text{taxable income}) \\
= \text{Earnings before taxes} - (\text{income tax rate}) \times (\text{earnings} - \text{depreciation})
\]

Define the symbols as follows:

- \( C \) = Cash flow, or the earnings after taxes, per year
- \( E \) = Earnings before taxes, per year
- \( t \) = Income tax rate
- \( D \) = Depreciation per year

Write the above result as

\[
C = E - t(E - D)
\]
Or, \[ C = E - tE + tD \]

Or, \[ C = E(1 - t) + tD \] (3.3)

We may write equation (3.1) as

\[ NPV = -I_0 + \sum_{i=1}^{n} \frac{E(1 - t) + tD}{(1 + r)^i} \] (3.4)

6. Resale, or residual value problems: Frequently there is a resale value of the project when it ends. For instance, a company may sell a used machine for cash. This is particularly important for assets that actually increase in value, such as land or buildings. After discounting, we add this final value to the cash flows. We must decide whether any taxes are due on this amount.

The taxes due on the sale depend upon the sale price and the book value of the asset. The book value of an asset is the depreciated value of the asset. In other words,

![Box](Book value of an asset = Initial cost of the asset minus the depreciation already taken)

A fully depreciated asset has no book value. The difference between the sale price and the book value is the capital gain on the sale. The tax on this additional income is the income tax rate multiplied by the capital gain. Thus

![Box](Tax due = Sale price minus Book value \times Income tax rate)

Write it in symbols, with \( B = \) book value, \( T = \) tax due, \( S = \) sale price, and \( t = \) income tax rate,

\[ T = (S - B)t \]

The after-tax cash from the sale is

\[ S - T = S - (S - B)t = S - St + Bt = S(1 - t) + Bt \]

After-tax cash from the sale of an asset = \( S(1 - t) + Bt \) (3.5)

If the book value is zero, that is, the asset is fully depreciated, then the after-tax proceeds are \( S(1 - t) \).

To illustrate the use of equation (3.5), consider the following example.

Suppose a company buys a machine for $5000, depreciates it uniformly in 5 years, and then sells it for $1000. Since the machine is depreciated fully after five years, its book value is zero. When the company sells it for $1000, there is a capital gain of $1000. If the income tax rate of the company is 30%, the company must pay \(.3(1000) = 300\) in taxes.
The after-tax proceeds of the sale is \(1000 - 300 = $700\). From (3.2), we get \(1000(1 - .3) = $700\).

In this example, the company is depreciating the machine on a straight-line basis at the rate of $1000 per year. Its book value after two years is $3000. If the company sells the machine after two years for $1500, its after-tax cash flow, according to (3.2) is \(1500(1 - .3) + 3000(.3) = $1950\).

A more detailed calculation is as follows. The company gets $1500 from the buyer of the used machine. The book value of the machine is $3000 and the company sells it for $1500, at a loss of $1500. It uses this capital loss as a deduction to offset other income. The tax benefit arising from this deduction is \(.3(1500) = $450\). The company gets a tax saving of $450. The total cash is $1500 + 450 = $1950, as calculated earlier.

7. **Unknown cash flows, or unknown life of the project.** In some cases, we do not know the cash flows from a certain project, and we must find these to make the project profitable. In other instances, we do not know how long a machine is going to last, and we must find the minimum number of years it must run to make the project worthwhile. We can find the unknown quantity by using (3.1), or (3.4).

**Examples**

**3.1. Basic:** Should we invest in a project that requires an initial outlay of $10,000, but will give back $1,000 annually for 20 years? We will receive the first payment after one year. The proper discount rate is 12%.

Use

\[
\text{NPV} = -I_0 + \sum_{i=1}^{n} \frac{C}{(1 + r)^i}
\]  

(3.1)

Put \(I_0 = 10,000\), \(C = 1000\), \(r = .12\), and \(n = 20\). This gives

\[
\text{NPV} = -10,000 + \sum_{i=1}^{20} \frac{1000}{1.12^i} = -10,000 + \frac{1000[1 - 1.12^{-20}]}{0.12} = - $2530.56.
\]

Since the NPV of the project is negative, we should reject it. ♥

Another way to look at the problem is that the return on a $10,000 investment, at the required rate of 12%, will be $1200 per year. Since it is generating only $1000 a year, it will never be profitable. If the discount rate were, say, 7\%, it would become profitable. We may see this as

\[
\text{NPV} = -10,000 + \sum_{i=1}^{20} \frac{1000}{1.07^i} = -10,000 + \frac{1000[1 - 1.07^{-20}]}{0.07} = $593.01
\]

To do the problem with Maple, we type in:

\[
\text{NPV} := -10000 + \text{sum}(1000/(1+r)^i, i=1..20);
\]
To do the problem with Excel, set it up as follows.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Initial investment, I $</td>
<td>10,000</td>
</tr>
<tr>
<td>2 Annual cash flow, C $</td>
<td>1000</td>
</tr>
<tr>
<td>3 Number of years, n</td>
<td>20</td>
</tr>
<tr>
<td>4 Discount rate, r</td>
<td>.12</td>
</tr>
<tr>
<td>5 NPV</td>
<td>(-B1+B2*(1-1/(1+B4)^B3)/B4)</td>
</tr>
</tbody>
</table>

This gives the desired results.

You may verify the answer at WolframAlpha by using this expressions,

\[
\text{WRA} -10000+\text{sum}(1000/1.12^i,i=1..20)
\]

\[
\text{WRA} -10000+\text{sum}(1000/1.07^i,i=1..20)
\]

### 3.2. Uneven cash flows

Find the NPV of a project whose initial cost is $100,000 and the proper discount rate is 12%. The project will generate $15,000 annually for years 1 through 5 and then $10,000 annually for the years 6 through 10. Should we accept the project?

We can set up the net present value as

\[
\text{NPV} = -100,000 + \sum_{i=1}^{5} \frac{15,000}{1.12^i} + \sum_{i=6}^{10} \frac{10,000}{1.12^i}
\]

Consider the second summation. Write it as

\[
\frac{10,000}{1.12^6} + \frac{10,000}{1.12^7} + \frac{10,000}{1.12^8} + \frac{10,000}{1.12^9} + \frac{10,000}{1.12^{10}}
\]

\[
= \frac{1}{1.12^5} \left[ \frac{10,000}{1.12} + \frac{10,000}{1.12^2} + \frac{10,000}{1.12^3} + \frac{10,000}{1.12^4} + \frac{10,000}{1.12^5} \right] = \frac{1}{1.12^5} \sum_{i=1}^{5} \frac{10,000}{1.12^i}
\]

Going back to the NPV equation,

\[
\text{NPV} = -100,000 + \sum_{i=1}^{5} \frac{15,000}{1.12^i} + \frac{1}{1.12^5} \sum_{i=1}^{5} \frac{10,000}{1.12^i}
\]

This simplifies to

\[
\text{NPV} = -100,000 + \frac{15,000(1 - 1.12^{-5})}{0.12} + \frac{1}{1.12^5} \left[ \frac{10,000(1 - 1.12^{-5})}{0.12} \right]
\]

\[
= -$25,474, \text{ reject it}
\]


You may verify the answer at WolframAlpha by using this expression,

\[-100000+\text{sum}(15000/1.12^i,i=1..5)+\text{sum}(10000/1.12^i,i=6..10)\]

To do the problem with Excel, set it up as follows.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Initial investment, $I_0$</td>
<td>$100,000</td>
</tr>
<tr>
<td>2</td>
<td>Annual cash flow, years 1-5, $</td>
<td>$15,000</td>
</tr>
<tr>
<td>3</td>
<td>Annual cash flow, years 6-10, $</td>
<td>$10,000</td>
</tr>
<tr>
<td>4</td>
<td>Discount rate, $r$</td>
<td>0.12</td>
</tr>
<tr>
<td>5</td>
<td>NPV</td>
<td>$=-B1+B2*(1-1/(1+B4)^5)/B4+B3*(1-1/(1+B4)^5)/B4/(1+B4)^5$</td>
</tr>
</tbody>
</table>

The Maple instruction for this problem is

\[\text{NPV}=-100000+\text{sum}(15000/1.12^i,i=1..5)+\text{sum}(10000/1.12^i,i=6..10);\]

### 3.3. Uneven cash flows:

You are considering a project with the following income stream: $10,000 annually for the first five years and then $8,000 annually for the next five years. The proper discount rate is 7%. How much should you invest in the project to break even?

To break even, the initial investment should be equal to the present value of all future income from the project. Thus

\[
P V \text{ of all future income} = \sum_{i=1}^{5} \frac{10,000}{1.07^i} + \sum_{i=6}^{10} \frac{8000}{1.07^i}
\]

\[
= \sum_{i=1}^{5} \frac{10,000}{1.07^i} + \frac{1}{1.07^5} \sum_{i=1}^{5} \frac{8000}{1.07^i}
\]

\[
= \frac{10,000(1-1.07^{-5})}{0.07} + \frac{1}{1.07^5} \left[ \frac{8000(1-1.07^{-5})}{0.07} \right] = $64,389
\]

To break even, the initial investment must also be $64,389.

You may verify the answer at WolframAlpha by using this expression,

\[\text{WRA sum}(10000/1.07^i,i=1..5)+\text{sum}(8000/1.07^i,i=6..10)\]

### 3.3. Uneven cash flows:

Find the NPV of a project that requires an initial investment of $10,000, another investment of $5,000 at the end of year 1, and gives a return of $20,000 at the end of year 3. The proper discount rate is 12%. Is the project acceptable?

An example of this kind of investment is if you buy a house for $10,000, renovate it for $5000, and then sell it for $20,000 after a couple of years. There are two negative cash
flows and a positive one. It is possible to represent the cash flows on a timeline as follows.

\[
\begin{array}{c|c|c|c|c}
0 & 1 & 2 & 3 \\
\downarrow -$10,000 & \downarrow -$5000 & \uparrow $20,000 \\
\end{array}
\]

The net present value of this investment is

\[
NPV = -10,000 - \frac{5,000}{1.12} + \frac{20,000}{1.12^3} = -$228.68, \text{ no, reject it.}
\]

You may verify the answer at WolframAlpha by using this expression,

\[
\text{WRA} -10000-5000/1.12+20000/1.12^3
\]

3.5. Uneven cash flows: An investment requires an initial outlay of $3,000, another expenditure of $3,500 at the end of the first year, but gives a return of $4,500 at the end of fourth year, and another return of $3,000 at the end of fifth year. If the proper discount rate is 6%, would you accept this investment?

Write the cash flows as

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$3000</td>
</tr>
<tr>
<td>1</td>
<td>-$3500</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>$4500</td>
</tr>
<tr>
<td>5</td>
<td>$3000</td>
</tr>
</tbody>
</table>

With a discount rate of 6%, find NPV as

\[
NPV = -3000 - \frac{3500}{1.06} + \frac{4500}{1.06^4} + \frac{3000}{1.06^5} = -$495.69. \text{ Reject it.}
\]

You may verify the answer at WolframAlpha by using this expression,

\[
\text{WRA} -3000-3500/1.06+4500/1.06^4+3000/1.06^5
\]

3.6. Uncertain cash flow: Ajax Corporation wants to set up an ice cream stand at an amusement park for the next 5 years. The initial investment in the project is $20,000. The ice cream sales will depend on the weather. The following table provides the subjective probability for different weather conditions and cash flows:

<table>
<thead>
<tr>
<th>Weather</th>
<th>Probability</th>
<th>Annual cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot</td>
<td>25%</td>
<td>$8,000</td>
</tr>
<tr>
<td>Average</td>
<td>60%</td>
<td>$5,000</td>
</tr>
<tr>
<td>Cool</td>
<td>15%</td>
<td>$3,000</td>
</tr>
</tbody>
</table>

Would you recommend setting up the shop if the discount rate is 8%?
First, calculate the expected annual cash flow by multiplying each probability by the outcome. This gives

\[ E(C) = 0.25 \times 8000 + 0.6 \times 5000 + 0.15 \times 3000 = 5,450 \]

Find the NPV as

\[ \text{NPV} = -20,000 + \sum_{i=1}^{5} \frac{5450}{1.08^i} = -20,000 + \frac{5450(1 - 1.08^{-5})}{0.08} = 1760.27 \]

The project is acceptable.

You may verify the answer at WolframAlpha by using this expression,

\[-20000 + \text{sum}(5450/1.08^i, i=1..5)\]

3.7. Taxes and depreciation: A firm is interested in buying a machine that costs $12,000 with a useful life of 6 years. It will generate pre-tax revenue of $4,000 for the company, which has a tax rate of 30%. The after-tax discount rate is 8%. Should the company buy the machine?

Assume straight-line depreciation. The amount of annual depreciation will be $12,000/6 = $2,000, because the machine does not have any residual value. The after-tax cash flow, \( C \), is given by (3.3)

\[ C = 4000(1 - 0.3) + 0.3(2000) = 3400 \]

From (3.1), we have

\[ \text{NPV} = -12,000 + \sum_{i=1}^{6} \frac{3400}{1.08^i} = -12,000 + \frac{3400(1 - 1.08^{-6})}{0.08} = 3717.79 \]

Yes, the company should buy the machine.

You may verify the answer at WolframAlpha by using this expression,

\[-12000 + \text{sum}(3400/1.08^i, i=1..6)\]

3.8. Taxes and depreciation: A firm has income tax rate of 40%, and it uses 7% discount rate. It needs a new machine costing $40,000 with a useful life of 5 years. It will generate pre-tax revenue of $10,000 annually. The company uses straight-line depreciation. Should the company buy the machine?

Using (3.3), find \( C = 10,000(1 - 0.4) + 0.4(40,000/5) = 9200 \)

\[ \text{NPV} = -40,000 + \sum_{i=1}^{5} \frac{9200}{1.07^i} = -40,000 + \frac{9200(1 - 1.07^{-5})}{0.07} = -2278.18 \]
Because of the negative NPV, the company should reject it.

You may verify the answer at WolframAlpha by using this expression,

\[WRA -40000+\sum(9200/1.07^i, i=1..5)\]

3.9. **Uncertain life of the project:** A project needs $24,000 to get started. It will give annual cash inflow of $6,000. The life of the project is uncertain: it may run for 5 years (probability 60%) or 6 years (probability 40%). If the proper discount rate is 10%, should we undertake the project?

*In this type of a problem, it is incorrect to start by first calculating the expected life of the project.* One should find the expected NPV by multiplying the probability of a given life by the dollar outcome of that life, and then adding together the results. We can do it as follows:

\[
\text{NPV} = -24,000 + 0.6 \left( \sum_{i=1}^{5} \frac{6000}{1.1^i} \right) + 0.4 \left( \sum_{i=1}^{6} \frac{6000}{1.1^i} \right)
\]

\[
= -24,000 + \frac{0.6(6000)[1 - 1.1^{-5}]}{0.1} + \frac{0.4(6000)[1 - 1.1^{-6}]}{0.1}
\]

\[
= $99.46, \text{ accept it.}
\]

You may verify the answer at WolframAlpha by using this expression,

\[WRA -24000+.6*\sum(6000/1.1^i,i=1..5)+.4*\sum(6000/1.1^i,i=1..6)\]

3.10. **Uncertain life of the project:** Rogers Corporation is planning to buy a machine for $24,000 that will add $6400 annually to the pre-tax income of the firm. The machine has an uncertain life: it may break down after 4 years (probability 30%) or 5 years (probability 70%). Using straight-line depreciation, the company will depreciate the machine completely in four years. The proper discount rate for the firm is 8% and its tax rate 28%. Should Rogers buy the machine?

The machine will definitely run for 4 years and it has a 70% chance for running through the fifth year. The depreciation is $6,000 annually. The depreciation lasts for 4 years and there is no tax benefit of depreciation in the fifth year. Using (3.3), the cash flow for the first four years, and for the fifth year, is

\[
C = 6400(1 - .28) + .28*6000 = $6288, \text{ for years 1-4}
\]

\[
C = 6400(1 - .28) = $4608, \text{ for year 5.}
\]

Thus, we have

\[
\text{NPV} = -24,000 + \sum_{i=1}^{4} \frac{6288}{1.08^i} + 0.7 \left( \frac{4608}{1.08^5} \right)
\]
\[
-24,000 + \frac{6288(1 - 1.08^{-4})}{.08} + 0.7 \left(\frac{4608}{1.08^5}\right) = -978.06
\]

The company should reject this machine. ♥

If you want to present your work in Excel, it may look like this. The following notes are required to explain your work fully:

The machine will definitely run for 4 years and it has a 70% chance for running through the fifth year. The depreciation is $6,000 annually. The depreciation lasts for 4 years and there is no tax benefit of depreciation in the fifth year. Using (3.3), the discounted cash flow for the years 0 through 5 are shown in row 9. Based on the negative NPV, which appears in cell B10, the company should not buy the machine.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Initial investment, $</td>
<td>24000</td>
<td>6000</td>
<td>6000</td>
<td>6000</td>
<td>6000</td>
</tr>
<tr>
<td>2</td>
<td>Depreciable Life, years</td>
<td>4</td>
<td>6400</td>
<td>6000</td>
<td>6000</td>
<td>6000</td>
</tr>
<tr>
<td>3</td>
<td>Earnings/year, E</td>
<td>.08</td>
<td>6000</td>
<td>6000</td>
<td>6000</td>
<td>6000</td>
</tr>
<tr>
<td>4</td>
<td>Year</td>
<td>.7</td>
<td>.7</td>
<td>.7</td>
<td>.7</td>
<td>.7</td>
</tr>
<tr>
<td>5</td>
<td>NPV</td>
<td>-SUM(B9:G9)</td>
<td>-SUM(B9:G9)</td>
<td>-SUM(B9:G9)</td>
<td>-SUM(B9:G9)</td>
<td>-SUM(B9:G9)</td>
</tr>
</tbody>
</table>

You may verify the answer at WolframAlpha by using this expression,

\[
-24000+\text{sum}(6288/1.08^i, i=1..4)+.7\times4608/1.08^5
\]

3.11. **Unknown life of the project**: A machine costs $32,000 and it will save a firm $3,000 annually. The proper discount rate for this project is 12%. The company is not paying any income tax. At least how many years must the machine run before it becomes profitable?

If a company is not paying any income taxes, its pre-tax and after-tax income is the same. To break even, the present value of the machine must be equal to the present value of earnings for n years:

\[
32,000 = \sum_{i=1}^{n} \frac{3000}{1.12^i} = \frac{3000(1 - 1.12^{-n})}{0.12}
\]

Or,

\[
32,000(.12)/3000 = (1 - 1.12^{-n})
\]
Or, 

\[ 1.28 - 1 = -1.12^{-n} \]

Take the ln of both sides,

\[ \ln(0.28) = n \ln(1.12) \]

Or,

\[ -1.272965676 = n \cdot 0.1133286853 \]

Or,

\[ n = -1.272965676 / 0.1133286853 = -11.23251075 \text{ years} \]

This is obviously not possible; a machine cannot run backward in time. There is no possible solution to the problem.

We look at the problem from another point of view; the machine will never become profitable no matter how long it runs. Suppose the machine is purchased by borrowing money at 12% interest rate, then the interest payments alone will be 0.12(32000) = $3,840 per year. The $3,000 generated by the machine will never cover its cost. In other words, the machine will never be profitable. ♥

You may verify the answer at WolframAlpha by using this expression,

\[ \text{WRA} \quad 32 = \text{sum}(3/1.12^i, i=1..x) \]

This gives the result \( x \approx 11.2325 - 277211i \), which is an imaginary number, meaning no real solution exists.

3.12. Unknown cash flow: Buckley College, a tax-exempt institution, is interested in acquiring a computer that costs $25,000 with a useful life of 5 years. Its resale value at that time will be $3000. The proper discount rate for the college is 9%. Find the minimum annual savings generated by the computer to justify its purchase.

Because of the tax-exempt status of the college, we do not have to worry about taxes or depreciation. Suppose the annual saving is \( S \). The resale value of the computer is $3000, available after 5 years. Its present value is \( 3000 / 1.09^5 \). To break even,

\[ \text{NPV} = 0 = -25,000 + \sum_{i=1}^{5} \frac{S}{1.09^i} + \frac{3000}{1.09^5} \]

Or,

\[ 25,000 - \frac{3000}{1.09^5} = \frac{S(1 - 1.09^{-5})}{0.09} \]

Or,

\[ 25,000 - 1949.794159 = 3.889651263 \times 3.889651263 \]

This gives \( S = \$5926.03 \) ♥

You may verify the answer at WolframAlpha by using this expression,

\[ \text{WRA} \quad 0 = -25000 + \text{sum}(S/1.09^i, i=1..5) + 3000/1.09^5 \]
3.13. *Unknown cash flow:* Bell Corporation is interested in buying a machine that costs $10,000. Bell will depreciate the machine on a straight-line basis over its 5-year useful life. The tax rate of Bell is 30%, and its discount rate is 14%. Find the minimum annual earnings generated by this machine to justify its purchase.

The depreciation per year is $2,000. Let the required minimum earnings before taxes be \( E \). The annual cash flow is

\[
C = E(1 - .3) + .3(2000) = .7E + 600
\]

To break even,

\[
NPV = 0 = -10,000 + \sum_{i=1}^{5} \frac{.7E + 600}{1.14^i}
\]

Or,

\[
10,000 = \frac{(.7E + 600)(1 - 1.14^{-5})}{.14}
\]

Or,

\[
.14(10,000) \frac{1}{1 - 1.14^{-5}} = .7E + 600
\]

Or,

\[
E = \frac{1}{.7} \left( \frac{.14(10,000)}{1 - 1.14^{-5}} - 600 \right) = 3,303.05
\]

The following spreadsheet also solves the problem.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Initial investment, $</td>
<td>10,000</td>
</tr>
<tr>
<td>2 Useful life, ( n ) years</td>
<td>5</td>
</tr>
<tr>
<td>3 Income tax rate, ( t )</td>
<td>.30</td>
</tr>
<tr>
<td>4 Discount rate, ( r )</td>
<td>.14</td>
</tr>
</tbody>
</table>

You may verify the answer at [WolframAlpha](https://www.wolframalpha.com) by using this expression,

\[
WRA \quad 0=-10000+\text{sum}((.7*x+600)/1.14^i,i=1..5)
\]

3.13. *Uncertain life:* Bernoulli Corporation is planning to acquire a machine that costs $20,000, but its life is uncertain. It may run for 4 years (probability 30%), or 5 years (probability 70%). The machine will create $4500 earnings before taxes per year. The proper discount rate for Bernoulli is 8% and the tax rate 30%. The depreciable life of the machine is 4 years. Should Bernoulli buy the machine?

The machine will be depreciated over four years, and thus the depreciation per year will be 20,000/4 = $5,000. The tax benefits are for the first four years only. Thus

\[
C = 4500(1 - .3) + .3*5000 = $4650, \text{ for years 1-4}
\]

\[
C = 4500(1 - .3) = $3,150, \text{ for year 5}
\]
The machine will definitely run for 4 years, whereas the probability of it running through the fifth year is 0.7. Thus the expected NPV is

\[
\text{NPV} = -20,000 + \sum_{i=1}^{4} \frac{4650}{1.08^i} + 0.7 \left( \frac{3150}{1.08^5} \right) 
\]

\[
= -20,000 + \frac{4650(1 - 1.08^{-4})}{.08} + 0.7 \left( \frac{3150}{1.08^5} \right) = -$3097.92
\]

The company should reject this proposal.

You may verify the answer at WolframAlpha by using this expression,

\[
WRA \ -20000+\sum(4650/1.08^i,i=1..4)+.7*3150/1.08^5
\]

### 3.15 Unknown cash flows: Fitzgerald Hospital, which is tax exempt, would like to buy a machine that costs $350,000, and is expected to last 7 years. The proper discount rate for the hospital is 7%. Find the minimum revenue from this machine to justify its purchase.

The hospital is tax-exempt, and so we do not consider taxes or depreciation. Assuming \(x\) to be the unknown revenue, we have,

\[
\text{NPV} = 0 = -350,000 + \sum_{i=1}^{7} \frac{x}{1.07^i}
\]

Or,

\[
350,000 = \frac{x(1 - 1.07^{-7})}{0.07}
\]

which gives

\[
x = \frac{0.07(350,000)}{1 - 1.07^{-7}} = $64,944 annually.
\]

You may verify the answer at WolframAlpha by using this expression,

\[
WRA \ 0=-350000+\sum(x/1.07^i,i=1..7)
\]

### 3.16 Resale value: Jamaica Airlines plans to buy an airplane for $30 million and depreciate it on a straight-line basis for 6 years, and then sell it for $12 million. While the airplane is flying, it will generate pre-tax earnings of $4 million annually. The income tax rate of Jamaica is 30%, and the appropriate discount rate is 10%. The airline has a choice of depreciating the plane in 6 years in one of the two ways. (A) It can depreciate it to its resale value. (B) It can depreciate it fully. Should the airline buy the plane?

(A) The airline chooses to depreciate the plane from its full value to its resale value. It will have total depreciation = 30 \(- 12 = $18 million over 6 years. The depreciation per year is thus $3 million. The cash flow per year is

\[
C = 4(1 - .3) + .3*3 = $3.7 million
\]
The company will sell the plane after 6 years creating $12 million in cash. There is no tax on this because the book value of the plane equals its sale price. Thus

\[ NPV = -30 + \sum_{i=1}^{6} \frac{3.7}{1.1^i} + \frac{12}{1.1^6} = -30 + \frac{3.7(1 - 1.1^{-6})}{.1} + \frac{12}{1.1^6} = -7.1118 \text{ million} \]

The airline should not buy the plane. ♥

You may verify the answer at [WolframAlpha](https://www.wolframalpha.com) by using this expression,

\[ \text{WRA} \ -30 + \text{sum}(3.7/1.1^i, i=1..6) + 12/1.1^6 \]

(B) If the airline decides to depreciate the plane completely, it will have get 30/6 = $5 million in depreciation annually. The cash flow in this case is

\[ C = 4(1 - .3) + .3 * 5 = $3.3 \text{ million}. \]

The plane is depreciated fully and its book value is zero. When the airline sells it for $12 million, this amount is taxable in full. Its after-tax value is 12(1 - .3) = $8.4 million. The NPV in this case is

\[ NPV = -30 + \sum_{i=1}^{6} \frac{4.3}{1.1^i} + \frac{8.4}{1.1^6} = -30 + \sum_{i=1}^{6} \frac{4.3(1 - 1.1^{-6})}{.1} + \frac{8.4}{1.1^6} = -6.5308 \text{ million} \]

It is still unacceptable. ♥

You may verify the answer at [WolframAlpha](https://www.wolframalpha.com) by using this expression,

\[ \text{WRA} \ -30 + \text{sum}(3.3/1.1^i, i=1..6) + 8.4/1.1^6 \]

3.17. **Resale value:** Cumberland Company wants to buy a machine for $40,000, depreciate it fully in four years using straight-line method, and then sell it for $5,000. The income tax rate of the company is 30% and its cost of capital 12%. The expected annual pretax earnings from the machine are $11,000. Should Cumberland buy the machine?

We can set up the problem as follows:

\[ C = E(1 - r) + tD = 11,000(1 - .3) + .3(10,000) = $10,700 \]

\[ \text{NPV} = -40,000 + \sum_{i=1}^{4} \frac{10,700}{1.12^i} + \frac{5000(1 - .3)}{1.12^4} \]

\[ = -40,000 + \frac{10,700(1 - 1.12^{-4})}{.12} + \frac{5000(1 - .3)}{1.12^4} = -5276.05, \text{ reject. ♥} \]
In an older textbook of finance, you may see the following approach to the above problem. We can also do these calculations on Excel. The discount factors are available in a table.

<table>
<thead>
<tr>
<th>Year</th>
<th>Income (expense)</th>
<th>Depreciation</th>
<th>Taxable income</th>
<th>Income taxes</th>
<th>After-tax income</th>
<th>Add back depreciation</th>
<th>Discount factor</th>
<th>Present value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(40,000)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>(40,000)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>11,000</td>
<td>10,000</td>
<td>1000</td>
<td>300</td>
<td>700</td>
<td>10,700</td>
<td>.8929</td>
<td>9553.57</td>
</tr>
<tr>
<td>2</td>
<td>11,000</td>
<td>10,000</td>
<td>1000</td>
<td>300</td>
<td>700</td>
<td>10,700</td>
<td>.7972</td>
<td>8529.97</td>
</tr>
<tr>
<td>3</td>
<td>11,000</td>
<td>10,000</td>
<td>1000</td>
<td>300</td>
<td>700</td>
<td>10,700</td>
<td>.7118</td>
<td>7616.05</td>
</tr>
<tr>
<td>4</td>
<td>11,000</td>
<td>10,000</td>
<td>1000</td>
<td>300</td>
<td>700</td>
<td>10,700</td>
<td>.6355</td>
<td>6800.04</td>
</tr>
<tr>
<td>4</td>
<td>5,000</td>
<td>5,000</td>
<td>1500</td>
<td>3500</td>
<td>3500</td>
<td>3500</td>
<td>.6355</td>
<td>2223.31</td>
</tr>
</tbody>
</table>

NPV = − 40,000 + 9553.57 + 8529.97 + 7616.05 + 6800.04 + 2223.31 = − $5276.06

The negative NPV makes the project unacceptable.

It is quite apparent that our current approach to the problems is efficient, convenient, and accurate.

You may verify the answer at WolframAlpha by using this expression,

\[ \text{WRA} = -40000 + \sum_{i=1}^{4} \frac{10700}{1.12^i} + 5000 \times \frac{(1 - .3)}{1.12^4} \]

**3.18. Resale value:** Aquarius Corporation needs a machine that costs $40,000, which will be used for 5 years, and after full depreciation, it will be sold for $8,000. The machine will have annual pretax earnings of $9,000. The tax rate of Aquarius is 30% and the cost of capital 11%. Should Aquarius buy the machine?

The cash flow for the first five years is 9000(1 − .3) + .3(8000) = $8700

After five years, the machine has zero book value. When it is sold for $8000, the whole amount is taxed as ordinary income at the rate of 30%, generating an after-tax cash flow of 8000(1 − .3) = $5600. Thus

\[ \text{NPV} = -40,000 + \sum_{i=1}^{5} \frac{8700}{1.11^i} + \frac{5600}{1.11^5} = -4,522.37, \text{ no.} \]

You may verify the answer at WolframAlpha by using this expression,

\[ \text{WRA} = -40000 + \sum_{i=1}^{4} \frac{8700}{1.11^i} + \frac{5600}{1.11^5} \]

**3.19. Unknown initial investment:** Carlson Corporation wants to buy a machine that would save them $2,000 before taxes per year. The company will depreciate the machine over its five-year life, without any resale value. The tax rate of Carlson is 30% and the
proper discount rate 12%. Find the maximum price that Carlson should pay for this machine.

Suppose the initial cost of the machine is $I_0$, then the relevant cash flow $C$ is:

$$C = 2000(1 - .3) + .3(I_0/5) = 1400 + .06 I_0$$

Equate the initial cost of the machine to the present value of subsequent cash flows:

$$I_0 = \sum_{i=1}^{5} \frac{1400 + .06I_0}{1.12^i}$$

Or,

$$I_0 = \sum_{i=1}^{5} \frac{1400}{1.12^i} + I_0 \sum_{i=1}^{5} \frac{.06}{1.12^i}$$

Or,

$$I_0 = 5046.69 + .216287 I_0$$

Or,

$$I_0 (1 - .216287) = 5046.69$$

Or,

$$I_0 = $6439.45$$

One can verify the result by sending these instructions to WolframAlpha. The highlighted part in the second line is the result of the first line.

WRA \(2000*(1-.3)+.3*x/5\)
WRA \(x=sum((.06*x+1400)/1.12^i, i=1..5)\)

3.20. Useful life more than depreciable life: Gaughan Corporation would like to buy a machine for $45,000 with a depreciable life of 5 year, with no resale value. The machine will actually run for 7 years and generate $10,000 in pretax revenue annually. The income tax rate of Gaughan is 31% and after-tax cost of capital 11%. Should Gaughan buy the machine?

The after-tax cost of capital is the appropriate discount rate. First, find the cash flows, using $C = E(1 - t) + tD$,

$$C = 10,000(1 - .31) + .31*9000 = 9690$$, for the first five years
$$C = 10,000(1 - .31) = 6900$$, for sixth and seventh year

Discounting and adding the cash flows, we find the NPV as

$$NPV = -45,000 + \sum_{i=1}^{5} \frac{9690}{1.11^i} + \frac{6900}{1.11^6} + \frac{6900}{1.11^7} = -$2173.29, no. \heartsuit$$

You may verify the answer at WolframAlpha by using this expression,
3.21. Uncertain life, depreciation, and taxes: Anderson Company is considering the purchase of a machine that will cost $12,000. Anderson will depreciate it on a straight-line basis over its useful life of 6 years. However, there is a 50% chance that it will break down after just 5 years. The machine will generate $2500 pretax earnings annually. The cost of capital is 10%, and its tax rate is 30%. Should Anderson buy the machine?

The machine will definitely run for 5 years. For the sixth year, either it will run, or it will not run, with equal probability. The cash flow from the machine while it is running is, \( C \), where

\[
C = E(1 - t) + tD = 2500(1 - 0.3) + 0.3 \times 2000 = 2350
\]

If the machine breaks down completely after 5 years, we take the tax benefit of the leftover depreciation, (.3)(2000) = $600 at that time. However, the probability of breakdown is 50%. The probability that it will run during the sixth year is also 50%. These two items generate the last two terms in the following expression:

\[
\text{NPV} = -12,000 + \sum_{i=1}^{5} \frac{2350}{1.1^i} + 0.5 \left( \frac{0.3 \times 2000}{1.1^5} \right) + 0.5 \left( \frac{2350}{1.1^6} \right)
\]

\[
= -12,000 + 8908.35 + 186.28 + 663.26 = -2242.12
\]

\[
\text{NPV} = -2242.12, \text{ no.}
\]

You may verify the answer at WolframAlpha by using this expression,

\[
-12000+\text{sum}(2350/1.1^i,i=1..5)+.5*.3*2000/1.1^5+.5*2350/1.1^6
\]

3.22. Unknown cash flow: Andromeda Corporation is planning to buy a new computer that will cost $68,000, with a useful life of 4 years. The tax rate of the company is 28% and its after-tax cost of capital is 9%. Find the minimum annual savings generated by the computer to justify its purchase.

Suppose the unknown annual savings is \( S \). The equivalent cash flow \( C \) is

\[
C = S(1 - .28) + .28(68,000/4) = .72 \times S + 4760
\]

To justify its purchase, the NPV of the computer should be at least zero,

\[
\text{NPV} = 0 = -68,000 + \sum_{i=1}^{4} \frac{.72 \times S + 4760}{1.09^i}
\]

Or,

\[
68,000 = (0.72 \times S + 4760)(1 - 1.09^{-4})/0.09
\]
Or, 
\[
\frac{0.09(68,000)}{1 - 1.09^{-4}} = 0.72S + 4760
\]

Or, 
\[
\frac{0.09(68,000)}{1 - 1.09^{-4}} - 4760 = 0.72S
\]

which gives \( S = 22,541 \)

The instructions for WolframAlpha are as follows. The highlighted part in the second line is the result of the first line.

\[
\text{WRA} \quad x*(1-.28)+.28*(68000/4)
\]
\[
\text{WRA} \quad 0=-68000+\text{sum}((.72x+4760)/1.09^i, i=1..4)
\]

3.23. **Uncertain life, depreciation and taxes:** Auriga Company wants to buy a truck for $25,000 with an expected life of 4 years (probability = 40%) or 5 years (probability = 60%). Auriga will depreciate the truck fully in 4 years. The savings produced by the truck will be $6000 a year. The tax rate of Auriga is 25% and its cost of capital 12%. Do you recommend the purchase of the truck?

Another way of expressing the probabilities is that the truck will definitely run for 4 years but it has only 60% probability of running during the fifth year. The cash flows are:

\[ C = 6000(1 - .25) + .25(25,000/4) = 6062.50 \text{, for the first 4 years} \]
\[ C = 6000(1 - .25) = 4500 \text{, for the fifth year} \]

\[
\text{NPV} = -25,000 + \sum_{i=1}^{4} \frac{6062.50}{1.12^i} + (0.6) \frac{4500}{1.12^5} = -5,053.02, \text{ no.} \]

You may verify the answer at WolframAlpha by using this expression,

\[
\text{WRA} \quad -25000+\text{sum}(6062.5/1.12^i, i=1..4)+.6*4500/1.12^5
\]

**Problems**

3.23. **Uneven cash flows:** A project has an initial investment of $22,000. It creates income of $3,000 annually for the first three years, and $5,000 for the next three years. If the proper discount rate is 8% annually, would you accept the project?

\[
\text{NPV} = -4,039.80, \text{ no}\]

3.25. **Uneven cash flows:** A project requires an initial outlay of $12,000, and gives a return of $1,000 annually for years 1-5 and then $2,000 annually for years 6-10. Is this project acceptable if the proper discount rate is 12%?

\[
\text{NPV} = -4,304, \text{ no.}\]
3.26. *Uneven cash flows*: Find the NPV of a project that requires an investment of $400 now; and another expense of $500 at the end of the first year. It gives cash inflows of $300 at the end of year 3, $400 at the end of year 4, and $800 at the end of year 5. The required rate of return is 11%. Is the project acceptable?  
NPV = $107, yes.

3.27. *Uneven cash flows*: What is the NPV of a project that requires an investment of $80,000 now, another $20,000 next year, but gives a return of $30,000 at the end of second year, $50,000 at the end of third year, and $40,000 at the end of fourth year. The discount rate is 12%. Should we invest in the project?  
NPV = $12,932, no.

3.28. *Uneven cash flows*: Find the NPV of a project that will involve an initial investment of $25,000, then another outlay of $10,000 after one year. It will give a cash flow of $6,000 annually for seven years, at the end of year 3 through year 9. The proper discount rate is 9%. Is it a profitable project?  
NPV = $8,757, no.

3.29. *Uneven cash flows*: A project will cost $45,000 to start, and will give a return of $10,000 at the end of first year, $12,000 at the end of second year, $15,000 at the end of third year, and $20,000 at the end of fourth year. The proper discount rate is 18%. Should we take this project?  
NPV = $8462, reject.

3.30. *Depreciation and taxes*: Atlas Corp needs a new machine that will cost $50,000. Using straight-line method, Atlas will depreciate it over its useful life of 5 years. The machine will add $14,000 annually to the earnings before interest and taxes (EBIT) of Atlas. The proper discount rate of Atlas is 12% and its tax rate is 32%. Should Atlas install the machine?  
NPV = $4147, no.

3.31. *Depreciation and taxes*: Lancaster Corp needs a new machine that will cost $60,000, and which will run for 5 years. The company uses straight-line depreciation. The machine will save $18,000 annually. The proper discount rate is 11%. The tax rate is 33%. Should Lancaster buy the machine?  
NPV = $791.73, no.

3.32. *Depreciation and taxes*: Quincy Corporation is in the 30% income-tax bracket. It needs a new computer costing $55,000, which will last for 5 years. Quincy uses straight-line basis depreciation. The computer will save the company $12,000 annually. The proper discount rate is 10%. Should Quincy buy the computer?  
NPV = $10,648, no.

3.33. *Depreciation and taxes*: Jones & Co. is considering the purchase of a machine that costs $40,000 and that will generate annual revenue of $13,000. Jones will depreciate it over its useful life of five years. If the tax rate of the company is 40%, and its proper discount rate 8%, should Jones buy the machine?  
NPV = $3,920, yes.

3.33. *Depreciation and taxes*: Ebsen Corp would like to buy a machine for $30,000 and depreciate it on a straight-line basis for 5 years. The pre-tax revenue from the machine is $8,000 per year, and the proper discount rate of Ebsen is 11%. If the income-tax rate of Ebsen is 30%, should it buy the machine?  
NPV = $2650, no.
3.35. **Depreciation and taxes:** Dryden Press wants to buy a new machine for $40,000 with a useful life of 5 years. Dryden uses straight-line method of depreciation. The machine will save the firm $10,000 annually. Should Dryden buy the machine if its tax rate is 38% and its after-tax discount rate 8%?  
NPV = − $3107, no ♥

3.36. **After-tax cash flows:** You have the opportunity to invest $10,000 in a project that will generate a pretax return of $4,000 annually for the next ten years. You are in the 28% tax bracket, and your after-tax required rate of return is 15%. Should you make the investment?  
NPV = $4454, yes ♥

3.37. **Resale value:** A machine has expected life of 6 years. It costs $30,000, and increases the net income of the company by $7,000 annually. It has a salvage value of $5,000. The proper discount rate is 12%, and the company pays no taxes at present. Would you recommend the purchase of this machine?  
NPV = $1,313, yes. ♥

3.38. **Uncertain life:** Mercy Hospital is planning to buy an x-ray machine whose total useful life is 4 years. However, there is a 25% chance that it may break down completely after 3 years. The machine will save $4,500 annually, and it will cost $11,000. The hospital is a tax-exempt entity, and its proper discount rate is 7%. Should Mercy buy the machine?  
NPV = $3,384, buy ♥

3.39. **Uncertain life:** A machine will run for 3 years (probability .35) or 4 years (probability .65). The annual revenue from the machine, before taxes, is $12,000. The proper discount rate in this case is 14%, and there are no taxes. The machine will cost $40,000. Should the firm buy the machine?  
NPV = − $7,522, no. ♥

3.40. **Uncertain life:** Alpha Corporation is planning to install a new computer that will cost $25,000. The computer will increase the after-tax earnings of the company by $6,000. The computer will become obsolete after 5 years (probability 30%) or after 6 years (probability 70%). The proper discount rate of Alpha is 12%. Should Alpha buy the computer?  
No, NPV = − $1,243 ♥

3.41. **Depreciation and taxes:** A machine costs $24,000 and it will run for 6 years. The machine will generate pre-tax annual revenue of $6,000 for the firm. The firm has tax rate 30% and after-tax discount rate 9%. Assuming straight-line depreciation, should the firm buy the machine?  
NPV = $224, buy it. ♥

3.42. **Uncertain life:** Eden Hospital is a tax exempt entity and it needs a NMR machine costing $250,000, which is expected to generate revenue of $60,000 annually. Its life will be either 4 years (probability 70%) or 5 years (probability 30%). If the proper discount rate to the hospital is 9%, should it buy the machine?  
NPV = − $43,918, no ♥

3.43. **Unknown cash flow:** Hammond Corp has income tax rate of 32% and its after-tax discount rate is 9%. Hammond plans to buy a machine for $53,000 that it will depreciate on a straight-line basis for 6 years with no salvage value. What is the minimum annual pretax income generated by this machine per year to justify its purchase?  
$13,218 ♥
3.43. **Uncertain life**: Kings Corp plans to buy a machine for $45,000 that will run for 4 years (probability 40%) or 5 years (probability 60%). The machine will produce a savings of $15,000 per year. The company does not pay any taxes and its discount rate is 12%. Should Kings buy this machine?

\[
\text{NPV} = 5,667, \text{yes}\]

3.45. **Resale value**: Ashraf Co needs a new machine that costs $40,000, which is expected to last for 4 years with $4000 salvage value, and which will add $13,000 annually to the EBIT of the company. The tax rate of Ashraf is 35% and it uses straight-line depreciation. The proper discount rate of Ashraf is 9%. Find if Ashraf should buy the machine under these assumptions:

(a) The machine is fully depreciated in 4 years, \[\text{NPV} = 556.56, \text{yes}\]
(b) The machine is depreciated to its resale value in 4 years, \[\text{NPV} = 413.45, \text{yes}\]

3.46. **Resale value**: Akmal Hospital is tax exempt. It would like to install a new computer costing $232,000 with a useful life of 8 years. The hospital plans to sell the computer for $20,000 after 5 years. The proper discount rate for the hospital is 9%. Find the minimum annual savings due to this computer to justify its purchase.

\[\text{NPV} = 56,304\]

3.47. **Unknown cash flows**: Emerson Corporation wants to install a new computer that costs $135,000, with an expected life of 6 years. The income tax rate of Emerson is 30%, it uses straight line depreciation, and its after tax cost of capital is 8%. Find the minimum annual savings created by the computer to justify its purchase.

\[\text{NPV} = 32,075\]

3.48. **Useful life more than depreciable life**: Kabul Co. needs a new machine that costs $120,000, which may run for 3 years (probability 35%) or 4 years (probability 65%). Kabul will depreciate it over a 3-year period. The machine is expected to produce an income of $40,000 a year while it is running. The tax rate of Kabul is 33% and its proper discount rate 12%. Should Kabul buy the machine?

\[\text{NPV} = -12,856, \text{no}\]

3.49. **Basic**: Cooper Company has an investment opportunity that requires an outlay of $10,000 now. It will give a return of $2,000 annually for 7 years, but the first payment will not be received until the end of the fifth year from now. The proper discount rate is 11%. Should Cooper make this investment?

\[\text{NPV} = -3,791.86, \text{no}\]

3.50. **Useful life more than depreciable life**: Burbank Company needs a new machine that costs $50,000. It will run for 6 years with no resale value. The company, however, will depreciate the machine fully in 5 years using straight-line method. The machine will generate $16,000 in pretax revenue annually. The cost of capital for Burbank is 11%, and its income tax rate 30%. Should it buy the machine?

\[\text{NPV} = 8470, \text{yes}\]

3.51. **Resale value**: Downey Company wants to buy a machine for $40,000, depreciate it fully in four years using straight line method, and then sell it for $8,000. The income tax rate of the company is 35%, and its cost of capital 12%. The expected annual pretax earnings from the machine are $10,000. Should Downey buy the machine?

\[\text{NPV} = -6322, \text{no}\]
3.52. *Useful life more than depreciable life*: Covington Company needs a new machine that costs $60,000 with a useful life of 6 years without any resale value. The company, however, will depreciate the machine fully in 5 years using straight-line method. The machine will generate $15,000 in pretax revenue annually. The cost of capital for Covington is 11%, and its income tax rate 35%. Should it buy the machine?

\[ \text{No, } \text{NPV} = -$3229 \]

3.53. *Depreciation and taxes*: Aberdeen Corporation is planning to buy a machine for $50,000 and depreciate it over its useful life of 5 years. While the machine is running, its pretax revenue will be $17,000 annually. The income tax rate of Aberdeen is 30%, and the discount rate for this investment is 12%. Should Aberdeen buy the machine?

\[ \text{Yes, } \text{NPV} = $3711 \]

3.53. *Uneven cash flows*: Dumbarton Company is interested in a project that will cost $40,000, and will run for 10 years. The cash flow from the project for the first five years is $6000 annually and for the next five years $5000 annually. The discount rate for the project is 11%. Should Dumbarton accept the project?

\[ \text{No, } \text{NPV} = -$6858 \]

3.55. *Unknown cash flows*: Falkirk Hospital is a tax-exempt entity. It wants to buy a new computer that will cost $225,000. Calculate the annual savings from the computer necessary to break even in 5 years. The cost of capital for the hospital is 8%.

\[ \text{$56,353} \]

3.56. *Resale value*: Starling Company wants to buy a tract of timberland for $200,000 and harvest trees. The net income from the timber is $25,000 annually for the next 5 years. Starling will then sell the land for $100,000. If the cost of capital is 10%, should Starling take this project? Starling does not pay income taxes.

\[ \text{No, } \text{NPV} = -$43,138 \]

3.57. *Depreciation and taxes*: Carlow Corporation is planning to buy a machine for $60,000 and depreciate it over its useful life of 5 years. While the machine is running its pretax revenue will be $17,000 annually. The income tax rate of Carlow is 30%, and the discount rate for this investment is 11%. Should Carlow buy the machine?

\[ \text{No, } \text{NPV} = -$2714 \]

3.58. *Uneven cash flows*: Cavan Company is interested in a project that will cost $25,000, and it will run for 8 years. The cash flow from the project for the first three years is $6000 annually, and for the next five years $5000 annually. The discount rate for the project is 11%. Should Cavan accept the project?

\[ \text{Yes, } \text{NPV} = $3174 \]

3.59. *Unknown cash flows*: Clare Hospital is a tax-exempt entity. It wants to buy a new computer that will cost $300,000. Calculate the annual savings from the computer necessary to break even in 6 years. The cost of capital for the hospital is 9%.

\[ \text{$66,876} \]

3.60. *Resale value*: Cork Company wants to buy a tract of timberland for $200,000 and harvest trees. The net income from the timber will be $25,000 annually for the first 4 years, and then $15,000 annually for the next 3 years. Cork expects to sell the land for
$150,000 after 7 years. Cork Company does not pay taxes. If the cost of capital is 10%, should Cork take this project? No, NPV = − $18,301♥

3.61. Unknown initial investment: Lagos College, a tax-exempt institution, needs a new computer that will produce annual savings of $13,000 for the next 10 years. The cost of capital for Lagos is 7%. Find the breakeven price for the computer. $91,307♥

3.62. Unknown initial investment: Gore Corporation wants to buy a machine that will save it $2,000 before taxes per year. This machine will last for 5 years and Gore will depreciate it over that period with no resale value. The tax rate of Gore is 30%, and its discount rate is 12%. Find the maximum price that Gore should pay for this machine. $6439♥

3.63. Depreciation and taxes: Weston Company needs a machine that is expected to run for 6 years and it will add $6,000 annually in pretax earnings. The cost of the machine is $24,000; the tax rate is 35%; and the proper discount rate is 10%. Weston will depreciate the machine uniformly over its life with no resale value. Should Weston buy the machine? No, NPV = − $917♥

3.63. Uncertain life, depreciation and taxes: Valley Corporation is considering the purchase of a machine that will run for either 5 years (probability 70%), or 6 years. Valley will depreciate it on a straight-line basis over 5 years. The cost of the machine is $60,000 and it will generate $15,000 annually in pretax earnings. The tax rate is 40%, and the discount rate is 12%. Should Valley buy the machine? No, NPV = − $8886♥

3.65. Uncertain life, depreciation and taxes: Dunedin Company is interested in buying a machine that may run for 6 years (40% probability) or 7 years (60% probability). The cost of the machine is $120,000. Dunedin will depreciate it on a straight-line basis over six years. There is no resale value. The machine will generate $30,000 annually in pretax earnings. The tax rate of Dunedin is 30% and its after-tax cost of capital 12%. Should Dunedin buy the machine? No, NPV = − $3292♥

3.66. Resale value: Hokitika Company is planning to buy a machine that will cost $100,000 and it will run for 5 years, and then it will be sold for $20,000. Hokitika will depreciate the machine in 5 years on a straight-line basis. The cost of capital for Hokitika is 11% and its tax rate 40%. The pretax income from the machine will be $20,000 annually. Should Hokitika buy the machine? No, NPV = − $18,961♥

3.67. Uncertain life, depreciation and taxes: Beaver Corporation wants to buy a machine that is expected to run for 5 years, but there is 20% chance that it may break down completely after 4 years. The value of the machine is $50,000 with a useful life of 5 years without any resale value. The machine will save the company $13,000 annually. The tax rate of Beaver is 30%, and the discount rate is 10%. Should Beaver buy the machine? No, NPV = − $5224♥
3.68. Resale value: Bedford College, a non-profit entity, wants to buy a computer and use it for the next 3 years. The computer will save the college $20,000 in the first year, $30,000 in the second year, and $40,000 in the third year. The initial cost of the computer is $120,000 and its resale value is $50,000. The cost of capital for Bedford is 8%. Should the college buy the computer? No, NPV = $−4316

3.69. Unknown cash flows: Akhisar Corporation would like to buy a machine for $80,000 with a useful life of 5 years with no resale value. Akhisar uses straight-line depreciation. The tax rate of Akhisar is 35% and the cost of capital 11%. Find the minimum annual savings generated by this machine to break even. $24,686

3.70. Depreciation and taxes: Gaziantep Company needs a new computer that will cost $30,000. It will run for 6 years, with no resale value. The company uses straight-line method of depreciation. The computer will save $6,000 annually. The cost of capital for Gaziantep is 12% and its income tax rate 40%. Should the company buy the computer? No, NPV = $−6976

3.71. Basic: Evans Company is going to buy 100 acres of land at $5,000 an acre. Evans will subdivide the land and sell it for $6,000 per acre, at the rate of 10 acres per year over the next 10 years. The cost of capital to the company is 15% per year. Is it a worthwhile project? No, NPV = $−198,873

3.72. Uncertain life: Fletcher Company will buy a machine for $50,000 that will create an annual income, after taxes, of $11,000. The cost of capital for Fletcher is 11%. The estimates of the life of the machine are 5 years (probability 30%), or 6 years (probability 70%). Should Fletcher buy the machine? No, NPV = $−5228

3.73. Resale value: A corporation would like to buy a machine for $50,000 that is expected to run for 6 years, and then it would be sold for $5,000. The company pays no taxes at present, and its cost of capital is 12%. Find the minimum earnings generated by the machine to justify its purchase. $11,545

3.73. Uncertain life, depreciation and taxes: Nelson Company needs a new machine that will cost $30,000. The machine will definitely run for 5 years, and it may even run for six years with a probability of 30%. Nelson will depreciate the machine on a straight-line basis for 5 years. The tax rate of Nelson is 30% and its after-tax cost of capital is 10%. The machine will create a savings of $6500 a year while it is operating. Should Nelson buy the machine? No, NPV = $−5158

3.75. Depreciation and taxes: Baxter Corporation has tax rate of 32% and its cost of capital is 13%. The company uses straight-line depreciation. It is considering the purchase of a machine for $10,000 that will add $2400 per year in pretax earnings. The machine will run for 5 years with no resale value. Should Baxter buy the machine? No, NPV = $−2009
3.76. Unknown cash flow: Baltimore College does not pay any taxes. The cost of capital for the college is 9%. At a cost of $70,000, the college wants to install a new computer that will run for 7 years. Just to break even, what must be the minimum annual saving from the computer? $13,908

3.77. Depreciation and taxes: Iceland Company is interested in buying a machine that will cost $50,000. The machine will have a depreciable life of 5 years. The machine will save $12,000 annually before taxes. The cost of capital for Iceland is 12%, and its income tax rate is 32%. Should Iceland buy the machine? No, NPV = $9050

3.78. Useful life more than depreciable life: Greenland Corporation needs a machine that costs $40,000. The company will depreciate it over 4 years using straight-line depreciation. The machine, however, has a useful life of 5 years during which it will generate $10,000 annually in pretax revenue. The cost of capital for Greenland is 11% and its income tax rate 30%. Should Greenland buy the machine? No, NPV = $4821

3.79. Depreciation and taxes: Sussex Corporation needs a new computer that costs $75,000. Sussex will depreciate it completely straight-line basis over 5 years. The computer will save the company $20,000 annually. The proper discount rate is 11%. The income tax rate for Sussex is 32%. Should it buy the computer? No, NPV = $6995

3.80. Unknown cash flow: Essex Company plans to buy a machine that will cost $65,000, which will be depreciated over 6 years to a resale value of $5,000. (Total depreciation available is $60,000, thus the depreciation is $10,000 a year.) The tax rate of the company is 28% and its cost of capital 12%. Calculate the minimum earnings before taxes per year generated by this machine to justify its purchase. $17,213

3.81. Uneven cash flows: Derbyshire Corporation plans to invest $25,000 in a project now, and another $20,000 by at the end of the first year. The project will have after-tax cash inflows of $7,000 a year for 8 years, the first one at the end of the third year. Is it a worthwhile project? The cost of capital for Derbyshire is 11%. No, NPV = $13,781

3.82. Resale value: Allen Corp is considering the purchase of a machine that costs $27,000. Allen will depreciate the machine uniformly for 5 years and then it will sell it for $2,000. The total available depreciation is thus $25,000. The yearly earnings before taxes from this machine are $6,000. The tax rate of Allen is 30%, and the proper discount rate is 12%. Should Allen buy the machine? No, NPV = $5318

3.83. Uncertain cash flows: Ball Corp needs a new printing press that costs $250,000 that will be depreciated uniformly over its expected life of 10 years. The estimates of its pretax annual earnings are $30,000 (Probability 25%), $40,000 (Probability 40%), and $50,000 (Probability 35%). The tax rate of Ball is 35% and its cost of capital 10%. Should Ball install the new press? No, NPV = $32,482

3.83. Depreciation and taxes: Noll Corp needs a new tractor that costs $25,000. Noll will depreciate it uniformly over its expected life of 10 years. The tractor will create pretax
annual earnings of $5,000. The tax rate of Noll is zero, and its cost of capital 10%. Should Noll buy the tractor? Yes, NPV = $5723 ♥

3.85. Resale value: Mellon Corp is considering the purchase of a machine that costs $56,000 and is expected to run for 5 years and then resold for $6,000. The total available depreciation is thus $50,000. The yearly earnings before taxes from this machine are $14,000. The tax rate of Mellon is 30% and its cost of capital 12%. Should Mellon buy the machine?
No, NPV = − $6454 ♥

Multiple Choice Questions

1. The discount rate of a project does not depend upon the  
A. risk of project,  
B. cost of capital of firm,  
C. prevailing interest rates,  
D. life of project.

3. The tax due at the time of the sale of an asset does not depend upon the  
A. income tax rate of firm,  
B. book value of asset,  
C. sale value of asset,  
D. discount rate for project.

2. The after-tax cash flow of a project does not depend upon the  
A. income tax rate of company,  
B. pre-tax earnings from project,  
C. riskless interest rate,  
D. annual depreciation of assets.

3. Define: \( T = \text{tax due at the time of sale of an asset, } t = \text{income tax rate, } B = \text{book value of the asset, } S = \text{its sale price.} \) The relationship between these quantities is  
A. \( T = (B - S)(1 - t) \)  
B. \( T = (S - B)(1 - t) \)  
C. \( T = (B - S)t \)  
D. \( T = (S - B)t \)

Key Terms

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