4. CAPITAL ASSET PRICING MODEL

Objectives: After reading this chapter, you should
1. Understand the use of beta as a measure of risk for a stock.
2. Understand and be able to apply the capital asset pricing model in financial modeling.

4.1 Beta

In the section on capital budgeting, we saw the need for a risk-adjusted discount rate for risky projects. The risk of an investment, or a project, is difficult to measure or quantify. This difficulty arises from the fact that different persons have different perceptions of risk. What may be quite a risky project to one investor may appear to be rather safe to another person. After all, how can you quantify courage, or patience, or risk, or beauty?

In the section on portfolio theory, we used the standard deviation of returns, \( \sigma \) as a measure of risk. Another useful measure of risk is the beta, \( \beta \) of an investment. Like \( \sigma \), \( \beta \) is also a statistical measure of risk. We calculate both these quantities from the observations of past performance of a stock. For example, we may want to find the risk of buying and holding the stock of a particular corporation, such as IBM, and we are interested in finding the \( \beta \) of IBM. We can start by looking at the historical value of three variables:

1. The returns of IBM stock, \( R_j \). We define the return on a stock by the relation

\[
R_j = \frac{P_1 - P_0 + D_1}{P_0}
\]  

(4.1)

In the above equation, \( P_0 \) is the purchase price of the stock, \( P_1 \) its price at the end of the holding period, and \( D_1 \) is the dividend paid, if any, at the end. The quantity \( P_1 - P_0 \) is the price appreciation of the stock. The quantity \( P_1 - P_0 + D_1 \) is the total change in the value of the investment. The return is equal to the change in the value of the investment divided by the original investment. For example, to find the monthly rate of return on the IBM stock, we may want to know the price of the stock at the beginning of each month, the price at the end of the month, and the dividends paid during that particular month. We have to develop a series of numbers representing the return for each month for the last 60 months, say.

2. The returns of the market, \( R_m \). We may judge the overall performance of the market by the value of a market index. The oldest and the most popular market index is the Dow Jones Industrial Average or DJIA. The problem with this index is that its value depends on the valuation of just 30 stocks. For a broader market index, we may have to look at S&P100, or S&P500 index. There is even an index for over-the-counter stocks called the NASDAQ Composite index. The value of these indexes is available daily.

Let us track the market for the last 60 months as well. If we know the value of the index at the start and finish of each month, we can find the return of the market for that month.
The dividend yield for the market is also available, and it is 2.51% annually at present (Second quarter, 2011). The overall return on the market is therefore,

\[ R_m = \frac{M_1 - M_0}{M_0} + d_t \]  

(4.2)

where \( M_0 \) and \( M_1 \) are the beginning and ending values of the market index, and \( d_t \) is the dividend yield as a percent for that period. With some effort, we may be able to develop a set of market returns for each of the last 60 months.

3. The riskless rate of interest, \( r \). The securities issued by the Federal government, such as the Treasury bills, bonds, and notes, are, by definition, riskless. They are the safest investments available, backed by the full faith and taxing power of the US government. Their rate of return depends on their time to maturity, and for longer maturity, the return is generally higher. The Treasury yield curve is readily available on the Internet.

After some research, we may also get a series of riskless rates, \( r \) for each of the past 60 months.

Then we define two variables \( x \) and \( y \) as:

\[ y = R_{j} - r \]
\[ x = R_m - r \]

where \( R_{j} \) = return on the stock \( j \) each month for the last 60 months,
\( R_m \) = corresponding monthly returns on the market for the same period,
and \( r \) = riskless rate of interest per month, for the last 60 months.

By subtracting the riskless rate of interest, we are able to see the return due to the risk inherent in the given stock, and the return from the risk in the market. Thus, we are comparing the returns exclusively due to the risk in the investments.

A regression line drawn between the observed values of \( x \) and \( y \) will show a certain linear relationship between them. The slope of the line will give the rate of change of \( y \) with respect to \( x \). In other words, the slope will signify how much the return on the stock will change corresponding to a given change in the return on the market. In this diagram let us say that the slope of the line is \( \beta \), and the \( y\)-intercept is \( \alpha \). The quantity \( \alpha \) is almost zero, and it is statistically insignificant. The quantity \( \beta \) represents an important concept.

We think of this responsiveness of the stock return to the changing market conditions as the "beta" of the stock. Stocks with low betas will show very little movement as the stock market moves up or down. High beta stocks will tend to be jumpy showing a large variation in response to small changes in the market.
High $\beta$ stocks, due to their large volatility, will be more unpredictable, and therefore, more risky. Low beta stocks show relatively small volatility, and they are more predictable and safe.

Beta is a statistical quantity, and it is a measure of the market related risk of a stock. We may also express these results as a statistical formula,

$$\beta_j = \frac{\text{cov}(R_j,R_m)}{\text{var}(R_m)}$$

(4.3)

where $\text{cov}(R_j,R_m)$ is the covariance between the returns on the stock $j$ and the market, and $\text{var}(R_m)$ is the variance of the returns on the market.

We can also use the concept of beta to measure the risk of a portfolio. The beta of a portfolio is simply the weighted average of the betas of the securities in the portfolio,

Beta of a portfolio, $\beta_p = w_1\beta_1 + w_2\beta_2 + w_3\beta_3 + \ldots = \sum_{i=1}^{n} w_i\beta_i$ (4.4)

The advantage of using $\beta$ as a measure of risk is that we can combine it linearly for different securities in a portfolio, but the disadvantage is that it can measure only the market related risk of a security. On the other hand, $\sigma$ can measure the risk that is independent of the market conditions, but its disadvantage is that it is non-linear in character and difficult to apply in practice. Note that $\sigma$ and $\beta$ are both incomplete measures of risk; they change with time; and are difficult to measure accurately.

The numerical value of $\beta$ for different stocks is available from sources such as www.yahoo.com, www.etrade.com, Value Line and Standard and Poors.
By definition, the beta of a riskless investment is zero. Also, by definition, the beta of the market is 1. This is seen by setting $j = m$ in (4.3) and noting that the covariance of a random variable with itself is just its variance.

### 4.2 Capital Asset Pricing Model

We are interested in developing a relationship between the risk and reward for a given investment. We want to find out how much risk one has to bear to achieve a certain rate of return. We have already seen that if you do not want to take any financial risk, you may still be able to earn the riskless rate of return. This is currently around 5 or 6 percent, depending on the time horizon of your investment. The question we are trying to answer is, how much additional risk gives rise to additional return.

![Security Market Line](image)

**Fig. 4.2:** The linear relationship between $\beta$ and $E(R)$, Security Market Line.

We may set up a coordinate system with $\beta$ along the $x$-axis and the expected return on the $y$-axis. The riskless asset is located at $(0, r)$, because its $\beta$ is zero and its return $r$. The investment in the market will be at $(1, E(R_m))$, because the $\beta$ of the market is 1, and its expected return is $E(R_m)$.

We have already noticed that $\beta$ is a linear measure of risk. If we assume that a linear relationship exists between the risk and return, then these two points are sufficient to draw a straight line in this diagram. The line is the *security market line*. Under equilibrium conditions, all other securities will also lie along this line. Higher $\beta$ securities will have a correspondingly higher expected return. Fig. 4.2 shows this.

The equation representing the security market line is (4.5):
Equation (4.5) represents what is known as the Capital Asset Pricing Model, or CAPM for short. The model was developed mostly by William Sharpe in 1964, and also by John Lintner, and by Jan Mossin. The model is a linear relationship between four quantities:

1. $E(R_j)$, the expected return on the security $j$,
2. $r$, the riskless rate of return,
3. $\beta_j$, the beta of the security $j$, and
4. $E(R_m)$, the expected return on the market.

We can apply this versatile model to a number of different financial problems.

### Examples

**4.1.** The following table gives information for the three states of the economy, the returns on the market, and the returns on the security $j$.

<table>
<thead>
<tr>
<th>State of economy</th>
<th>Probability</th>
<th>Return on market</th>
<th>Return on $j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>0.2</td>
<td>0.4</td>
<td>0.485</td>
</tr>
<tr>
<td>Fair</td>
<td>0.5</td>
<td>0.2</td>
<td>0.235</td>
</tr>
<tr>
<td>Poor</td>
<td>0.3</td>
<td>−0.1</td>
<td>−0.14</td>
</tr>
</tbody>
</table>

Find the following:

(A) The expected return of the market
(B) The expected return on the security $j$
(C) The variance, and $\sigma$, of market returns
(D) The variance, and $\sigma$, of security $j$
(E) The covariance and the correlation coefficient between the market and the security $j$
(F) The $\beta$ of security $j$
(G) The riskless interest rate

(A) To find the expected return of the market, we multiply each probability with each outcome, and then add them. Using (1.8), we have

$$E(R_m) = 0.2(0.4) + 0.5(0.2) + 0.3(−0.1) = 0.15$$

(B) Similarly, $E(R_j) = 0.2(0.485) + 0.5(0.235) + 0.3(−0.14) = 0.1725$

(C) To find the variance and sigma of the market, we use (1.9) and (1.10), which give

$$\text{var}(R_m) = 0.2(0.4 - 0.15)^2 + 0.5(0.2 - 0.15)^2 + 0.3(−0.1 − 0.15)^2 = 0.0325$$
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4. Capital Asset Pricing Model

and \( \sigma_m = \sqrt{0.0325} = 0.1803 \)

(D) In a similar manner, we find

\[
\text{var}(R_j) = 0.2(0.485 - 0.1725)^2 + 0.5(0.235 - 0.1725)^2 + 0.3(-0.14 - 0.1725)^2
\]

\[
= 0.05078 \quad \text{\( \sigma_j = \sqrt{0.05078} = 0.2253 \)}
\]

(E) To find the covariance, we use (5.4). This gives us

\[
\text{cov}(R_m, R_j) = 0.2(0.4 - 0.15)(0.485 - 0.1725) + 0.5(0.2 - 0.15)(0.235 - 0.1725)
\]

\[
+ 0.3(-0.1 - 0.15)(-0.14 - 0.1725) = 0.040625 \quad \text{\( R_{mj} = \frac{0.040625}{0.1803(0.2253)} = 1 \)}
\]

The correlation coefficient is 1 because the return on the stock is perfectly correlated with the return on the market. We may demonstrate this by using CAPM. In the first line in the table, if the expected market return is 40\%, then the stock return is

\[
E(R_j) = .06 + 1.25(.4 - .06) = .485,
\]

as expected. For the second and the third lines, we get

\[
E(R_j) = .06 + 1.25(.2 - .06) = .235,
\]

\[
E(R_j) = .06 + 1.25(-.1 - .06) = -.14.
\]

(F) To find the \( \beta \) of the stock, we use (4.3).

\[
\beta_j = 0.040625/0.0325 = 1.25 \quad \text{\( W \)}
\]

We notice that the \( \beta \) of the stock is more than 1. This is because the return on the stock changes somewhat more than the changes in the market. The estimate for the market high and low is 40\% and -10\%, whereas the stock's high and low are 48.5\% and -14\%.

(G) To find the riskless interest rate we use the CAPM,

\[
E(R_j) = r + \beta_j [E(R_m) - r] \quad (4.5)
\]

which gives

\[
0.1725 = r + 1.25[0.15 - r]
\]

We get \( r = .06 \)
4.2. The beta of KLM stock is 1.22, the expected market return is 15%, and the riskless rate is 9%. If you invest $12,000 in KLM stock now, what is the expected value of your investment after 3 years, assuming annual compounding?

Using CAPM, we find the expected return on the stock to be

\[ E(R) = 0.09 + 1.22 (0.15 - 0.09) = 0.1632 \]

With a 16.32% growth rate, the future value of the stock after three years is

\[ FV = 12,000(1.1632)^3 = $18,886 \]

4.3. Beaver Corp stock has beta of 1.25 and the one-year T-bill rate is 6%. The probability distribution on the market returns is as follows. Find the expected return on Beaver stock.

<table>
<thead>
<tr>
<th>Probability</th>
<th>Market return</th>
</tr>
</thead>
<tbody>
<tr>
<td>40%</td>
<td>30%</td>
</tr>
<tr>
<td>30%</td>
<td>10%</td>
</tr>
<tr>
<td>30%</td>
<td>-5%</td>
</tr>
</tbody>
</table>

First, we find the expected return on the market, which is

\[ E(R_m) = 0.4(0.3) + 0.3(0.1) + 0.3(-.05) = 0.135 \]

Using CAPM, we get the return on the stock to be

\[ E(R_j) = 0.06 + 1.25(0.135 - 0.06) = 0.15375 = 15.375\% \]

4.4. The covariance between the return of the market and the return of Abel Company stock is .0182, whereas the variance of return of market is 0.0125. The expected return of market is 10%, the riskless rate is 6%, and the expected return of Abel Company is 12%. Find two independent values of beta of Abel.

We can find one value of \( \beta \) by using (4.3). This gives us

\[ \beta_1 = \frac{\text{cov}(R_j, R_m)}{\text{var}(R_m)} = \frac{0.0182}{0.0125} = 1.456 \]

From (4.5),

\[ E(R_j) = r + \beta_j [E(R_m) - r] \]

we get

\[ .12 = .06 + \beta_2 [.1 - .06] \]

Or,

\[ \beta_2 = (0.12 - 0.06)/(0.10 - 0.06) = 1.5 \]

The two answers are quite similar. After all, it is difficult to find the value of \( \beta \) accurately.
4.5. Find the β of a stock that gives a 5% dividend return and expects to grow 10% annually. The return on the market is 14% and the riskless rate is 9%.

\[ E(R) = \text{dividend yield} + \text{expected growth rate} = 0.05 + 0.1 = 0.15, \]

We also know that \( E(R_m) = 0.14 \), and \( r = 0.09 \). Then by CAPM,

\[ 0.15 = 0.09 + \beta(0.14 - 0.09) \]

which gives

\[ \beta = 0.06/0.05 = 1.2 \]

4.6. The beta of Texaco is 1.21, it will pay a dividend of $3 next year, and its current price is $40. Investors expect Texaco to grow at the rate of 5.5% in the long-run. The riskless rate is currently 8%. Find the expected return on the market.

Using Gordon's growth model, (2.7)

\[ P_0 = \frac{D_1}{R - g} \]

We write the required return, \( R \), as

\[ R = \frac{D_1}{P_0} + g \]

Substituting number, we get

\[ R = 3/40 + 0.055 = 0.13 \]

Next, use CAPM, and equate it to the expected return \( E(R) \),

\[ .13 = 0.08 + 1.21[E(R_m) - 0.08], \]

which gives \( E(R_m) = (0.13 - 0.08)/1.21 + 0.08 = 0.1213 = 12.13\% \)

4.7. Alcoa stock is selling at $40 and it just paid the annual dividend of $1.20. Investors expect that Alcoa will grow at the annual rate of 12%. The expected return on the market is 13% and the riskless rate is 7%. Find the beta of Alcoa.

By Gordon's growth model, (2.7), we have

\[ R = \frac{D_1}{P_0} + g \]

Next year’s dividend, \( D_1 \) is 12% higher than this year’s. Thus \( D_1 = 1.12(1.2) = $1.344 \).

The required rate of return, \( R = 1.344/40 + 0.12 = 0.1536 \)

Using CAPM, (4.5),

\[ E(R_j) = r + \beta_j [E(R_m) - r] \]
we have
\[ 0.1536 = 0.07 + \beta(0.13 - 0.07) \]
which gives
\[ \beta = 1.393 \]
To verify the answer on Excel, you can set up the following table.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Price of the stock, ( P_0 ) =</td>
</tr>
<tr>
<td>2</td>
<td>Current dividend = ( D_0 ) =</td>
</tr>
<tr>
<td>3</td>
<td>Growth rate of stock, ( g ) =</td>
</tr>
<tr>
<td>4</td>
<td>Expected return on market, ( E(R_m) ) =</td>
</tr>
<tr>
<td>5</td>
<td>Riskless rate, ( r ) =</td>
</tr>
<tr>
<td>6</td>
<td>Required rate of return, (GGM), ( R ) =</td>
</tr>
<tr>
<td>7</td>
<td>Expected rate of return (CAPM), ( E(R) ) =</td>
</tr>
<tr>
<td>8</td>
<td>( \beta ) of stock =</td>
</tr>
</tbody>
</table>

4.8. The price of Bucks Corp stock is $40 per share. It has a beta of 1.24, and it will pay a dividend of $3 next year. The expected return on the market is 12%, and the riskless rate is 6%. What is the long-term growth rate of Bucks?

From CAPM, we have
\[ E(R) = 0.06 + 1.24(0.12 - 0.06) = 0.1344. \]

From Gordon's growth model, (2.7),
\[ R = \frac{D_1}{P_0} + g \]
which gives,
\[ g = R - \frac{D_1}{P_0} = 0.1344 - 3/40 = 0.0594 = 5.94\% \]

Problems

4.9. You should look up the \( \beta \) of the following stocks. You will notice that \( \beta \) for the same stock is different as reported by different agencies.

<table>
<thead>
<tr>
<th>Stock</th>
<th>Source 1</th>
<th>( \beta_1 )</th>
<th>Source 2</th>
<th>( \beta_2 )</th>
<th>Average ( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dell</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IBM</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Merck</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pfizer</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Target</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wal-Mart</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.10. Knoll Inc stock has \( \beta = 1.25 \), the expected return on the market is 12%, and the riskless rate is 8%. Knoll Inc may grow annually at the rate of 3% for many, many years. Next year Knoll will pay a dividend of $3.00. Find the price of Knoll stock.

\[ \$30.00 \]

4.11. You have developed the following data for the three states of the economy, the returns on the market, and the returns on the security \( j \). Find the beta of security \( j \).
### State of economy  
<table>
<thead>
<tr>
<th>Probability</th>
<th>Return on market</th>
<th>Return on j</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>0.2</td>
<td>0.15</td>
</tr>
<tr>
<td>Fair</td>
<td>0.4</td>
<td>0.10</td>
</tr>
<tr>
<td>Poor</td>
<td>0.4</td>
<td>−0.05</td>
</tr>
</tbody>
</table>

$$\beta = 0.6571$$

**4.12.** The stockholders of Gordon Company expect a return of 14% and its $\beta$ is 1.2. If the one-year Treasury bill rate is 8%, find the expected return on the market.  

**13%**

**4.13.** The risk of IBM is exactly that of the market. IBM will pay a dividend of $4.40 next year and $4.60 two years from now, which is in line with its long-term growth record. IBM stock sells at $120 a share. Find the expected return on the market.  

**8.21%**

**4.14.** Cornfeld Company stock has beta of 1.1 and its expected return is 12%, whereas Goldstein Company stock has beta of 1.2 and its expected return is 12.3%. Find the riskless rate and the expected return on the market.  

$$r = 8.7\%, \ E(R_m) = 11.7\%$$

**4.15.** Ajmal Company has beta of 1.25, growth rate of 4.5%, and dividend next year of $1.25. Ajmal stock sells at $11 a share and the riskless rate is 9%. Find the expected return on the market.  

**14.49%**

**4.16.** The expected return on the market next year is 15%, whereas the riskless rate is 6%. The beta of IBM is 0.8, its current price is $120 per share, and its dividend next year is $6. Using CAPM, find the expected price of IBM stock next year.  

**$129.84**

**4.17.** You want to invest $10,000 in a stock with beta 1.25. Currently the riskless rate is 9% and the expected return on the market is 12%. What is the expected total dollar value of your investment after two years?  

**$12,713**

**4.18.** The beta of Ford stock is 1.22; its dividend next year is $4.00, which is expected to grow at the rate of 5% for a long time. The price of Ford stock is $42. The riskless rate is 7%. Find the expected return on the market.  

**13.16%**

**4.19.** The price of a share of ExxonMobil stock is $60 today. It will pay a dividend of $3 next year and its total return should be 12%. Well-informed economists expect the return on the market to be 10% and the riskless rate is 6.5%. Find the beta and the expected price of a share of ExxonMobil stock next year.  

$$\beta = 1.57, \$64.20$$

**4.20.** Born Company stock is selling for $50 a share and its beta is 1.25. The expected return on the market is 12% whereas the riskless rate is 6%. What is the return on Born stock in dollars per share?  

**$6.75**

**4.21.** The $\beta$ of Faulkner Corp is 1.33 whereas the riskless rate is 7%. The expected return of the market is 13%. If Faulkner shares are selling for $66 each and they do not pay any dividend, what is their expected price after one year?  

**$75.89**
**4.22.** Hess Oil Company stock sells for $25 a share. It has a growth rate of 8%, and its dividend next year will be $1.25. If the expected return on the market is 12% and the riskless rate is 8%, find the β of Hess.

**4.23.** Frost Corp has β = 1.24, and growth rate of 4%. It just paid its annual dividend of $4 and the riskless rate is 6%. If the expected return on the market is 13%, find the price of a share of Frost stock.

**4.24.** You have invested $22,000 in Jenkins Company stock, which has a β = 1.3. You expect that the market will rise by 10% this year and only 8% next year. The risk-free interest rate will remain constant at 3%. Find the expected value of your investment after two years.

**4.25.** The risk-free interest rate, \( r \) and the expected return of the market, \( E(R_m) \) depend on the action of the Federal Reserve, which meets tomorrow. They may increase the interest rates or leave them unchanged.

<table>
<thead>
<tr>
<th>Interest rates</th>
<th>Probability</th>
<th>( r )</th>
<th>( E(R_m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Will increase</td>
<td>30%</td>
<td>4%</td>
<td>10%</td>
</tr>
<tr>
<td>Remain unchanged</td>
<td>70%</td>
<td>3%</td>
<td>12%</td>
</tr>
</tbody>
</table>

Bailey Corporation stock is selling at $50 per share now and its β is 1.2. Find the expected price of the stock after one year.

**Multiple-choice Questions**

1. The β of stock is not

   A. A statistical quantity.
   B. A measure of its market related risk.
   C. Equal to 1 for an average-risk stock.
   D. Related to the price of the stock.

2. According to CAPM,

   A. β is a nonlinear measure of risk.
   B. All securities must lie below the security market line.
   C. Higher risk stocks should provide lower returns in the long run.
   D. The expected return of the market is more than the riskless rate of return.

**Key terms**

- Dividend yield, 67, 73
- DJIA, 66
- NASDAQ, 66
- Return, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76
- S&P100, 66
- S&P500, 66
- Risk, 66, 67, 68, 69, 75, 76