MATH 361, Chapter 4 Homework
May 3, 2013

Directions: Solution to this is due with the other homework at the start of class on Friday, May 10. Remember unsupported answers may receive no credit.

1. Let \( f(x) = x^2 e^{-x} \).

(a) Find the polynomial interpolant over \([0, 3]\) using nodes \( x_k = k, \ k = 0, 1, 2, 3 \).

(b) Find the clamped spline that interpolates \( f \) over \([0, 3]\) using nodes \( x_k = k, \ k = 0, 1, 2, 3 \) in two ways: (i) using the full system to solve for the cubic coefficients \( a_i, b_i, c_i, \) and \( d_i \) and (ii) using the efficient method to solve for the \( \sigma \)'s. In each case, your work must include the system (either in equation form or in matrix form) that must be solved, but you may use MATLAB to solve. You must also write out the cubics explicitly.

(c) Find an upper bound on the error of the clamped spline as an approximation to \( f \).

(d) Plot on the same axes \( f \), the clamped spline, and the polynomial interpolant found above over \([0, 3]\) and comment.

(e) Extra credit: Estimate an interval in which the maximum error exists between the clamped spline and \( f \) and find the point of maximum error using a root-finding method of your choice. What is the actual maximum error? (Hint: You may find it helpful to look at the graph of \(|f(x) - p(x)|\) where \( p \) is the cubic spline to \( f \).)

2. For extra credit repeat the above with \( f(x) = 1/(1 + x^2) \), a modified version of the Runge function, on \([-4, 4]\) with nodes \( x_0 = -4, x_1 = -2, x_2 = 0, x_3 = 2, x_4 = 4 \). You may simply use the \( \sigma \)'s to find the spline.